

Truth values  $\Rightarrow$

if $1+1=3$ then dogs can fly	<del>False</del> $F \rightarrow F \equiv T$
if $2+2=4$ then today is Monday	$T \rightarrow F \equiv T$
if $2+4=6$ then $1+2=3$	$T \rightarrow T \equiv T$
$2+2=5$ iff $3+3=5$	$F \leftrightarrow F \equiv T$
dogs can fly iff $2+2=4$	$F \leftrightarrow T \equiv F$

Equivalence	Name
$P \wedge T \equiv P$ $P \vee F \equiv P$	Identity law
$P \vee T \equiv T$ $P \vee F \equiv P$	Domination law
$P \vee P \equiv P$ $P \wedge P \equiv P$	Idempotent laws
$\neg(\neg P) \equiv P$	Double negation law
$P \wedge q \equiv q \wedge P$ $P \vee q \equiv q \vee P$	Commutative law
$(P \vee q) \vee r \equiv P \vee (q \vee r)$ $(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$	Associate law

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

Distributive law

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

De Morgan's Law

$$P \vee (P \wedge Q) \equiv P$$

$$P \wedge (P \vee Q) \equiv P$$

Absorption Law

$$P \vee \neg P \equiv T$$

$$P \wedge \neg P \equiv F$$

Negation Law

$$\textcircled{1} P \rightarrow Q \equiv \neg P \vee Q$$

$$\textcircled{2} P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$$

Q.  $P \rightarrow Q$  and  $P \wedge \neg Q$

Sol.

$$\neg(P \rightarrow Q)$$

$$\neg(\neg P \vee Q)$$

$$\neg \neg P \wedge \neg Q$$

$$P \wedge \neg Q$$

proved

$\therefore$  Demorgan law

$\therefore$  Double Negation law.

Q2:

$$\neg(P \vee (\neg P \wedge q)) \text{ and } \neg P \wedge \neg q$$

Sol: L.H.S

$$\begin{aligned} & \neg(P \vee (\neg P \wedge q)) \\ \equiv & \neg(P \wedge \neg(\neg P \wedge q)) && \because \text{De Morgan law} \\ \equiv & \neg P \wedge (\neg \neg P \vee \neg q) && \because \text{"} \\ \equiv & \neg P \wedge (P \vee \neg q) && \text{Double negation} \\ \equiv & (\neg P \wedge P) \vee (\neg P \wedge \neg q) && \because \text{Distribution} \\ = & F \vee (\neg P \wedge \neg q) && \because \text{Negation} \\ \equiv & \neg P \wedge \neg q && \because \text{Identity} \end{aligned}$$