12/6/15

(Lec no 15, 16)

Linear Differential Equation

Definition:-

A first order D.E is of form

$$\frac{dY}{dx} + P_{(x)}Y = Q_{(x)}$$

is said to be linear, if

i) Dependent ~~[illegible]~~ variable 'y' and its derivatives is of 1 degree.

ii) The co-efficient of dependent variables and its derivatives is at most of in-dependent variables.

→ Linear D.E has two types:-

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Homogeneous L.D.E	Non-Homogeneous L.D.E
→ $\frac{dY}{dx} + P_{(x)}Y = 0$	$\frac{dY}{dx} + P_{(x)}Y = Q_{(x)}$
→ Seperable D.E	

Non-Homogeneous L.D.E

Solution.-

$$dY/dx + P_{(x)}Y = Q_{(x)}$$

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Step 1:- If multiplies the whole equation with variable (V_x). So that the left hand side will become differential derivative of $(V_x \cdot Y)$

$$V_x \frac{dy}{dx} + P_{(x)} V_{(x)} Y = Q_{(x)} V_{(x)}$$

$$\frac{d}{dx}(V_x \cdot Y) = Q_{(x)} V_{(x)}$$

$V_{(x)}$ is called Integrating Factor.

Step 2 - Now Integrate it.

$$\int \frac{d}{dx}(V_{(x)} \cdot Y)\, dx = \int Q_{(x)} V_{(x)}\, dx$$

$$V_{(x)} \cdot Y = \int Q_{(x)} V_{(x)}\, dx$$

$$\boxed{Y = \frac{1}{V_{(x)}} \int Q_{(x)} V_{(x)}\, dx}$$

→ How To Calculate Integrating Factor of Non-Homogeneous LDE :-

$$dy/dx + P_{(x)} Y = Q_{(x)}$$

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Solution:-

$$V_{(x)} \frac{dY}{dx} + V_{(x)} P_{(x)} Y = Q_{(x)} V_{(x)}$$

Solve L.H.S:-

$$V_{(x)} dY/dx + P_{(x)} V_{(x)} y = \frac{d}{dx} (V_{(x)} Y)$$

$$V_{(x)} \cancel{dY/dx} + P_{(x)} V_{(x)} Y = V_{(x)} \cancel{dY/dx} + Y \frac{d V_{(x)}}{dx}$$

$$P_{(x)} V_{(x)} \cancel{Y} = \cancel{Y} \frac{d(V_{(x)})}{dx}$$

$$\frac{dV}{dx} = P_{(x)} V_{(x)}$$

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$$\frac{dV}{V} = P_x dx$$

Integrate it,

$$\int \frac{dV}{V} = \int P_{(x)} dx$$

$$\ln V = \int P_{(x)} dx$$

$$\boxed{V = e^{\int P_{(x)} dx}} = \text{Integrating Factor}$$

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Practice Questions

Question no 1:-

$$x \frac{dy}{dx} = x^2 + 3y$$

Dividing by x

$$dy/dx = x + 3y/x$$

$$\frac{dy}{dx} + \left(-\frac{3}{x}y\right) = x \quad \text{——— (i)}$$

$$P_{(x)} = -3/x$$

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So,

$$V_{(x)} = e^{\int(-3/x)dx}$$

$$= e^{-3\ln x} = e^{\ln x^{-3}}$$

$$I.F = \boxed{V_{(x)} = x^{-3}}$$

Multiply I.F with eq (i)

$$x^{-3}\, dy/dx - (3/x)(x^{-3})(y) = x(x^{-3})$$

$$\frac{d}{dx}(x^{-3} \cdot y) = x^{-2}$$

Integrate b/s,

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$$\int \frac{d}{dx}(x^{-3}y)\,dx = \int x^{-2}\,dx$$

$$x^{-3}y = \left(\frac{x^{-2+1}}{-2+1}\right) + c$$

$$x^{-3}y = -x^{-1} + c$$

Dividing by x^{-3},

$$\boxed{y = \frac{-x^{-1}}{x^{-3}} + \frac{c}{x^{-3}}}$$ General Solution

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Question no 2:-

$$dy/dx + y\tan x = \sin 2x \quad \text{——— (i)}$$

$$P_{(x)} = \tan x$$

$$V_{(x)} = e^{\int \tan x\,dx}$$

$$V_{(x)} = e^{-\int -\frac{\sin x}{\cos x}\,dx}$$

$$V_{(x)} = e^{-\ln \cos x}$$

$$V_{(x)} = \cos x^{-1} = \sec x$$

multiply $V_{(x)}$ with eq (i)

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$$\sec x \, dy/dx + y(\sec x)(\tan x) = \sec x \sin 2x$$

$$\frac{d}{dx}(\sec x \cdot y) = \frac{1}{\cos x}(2 \sin x \cos x)$$

$$\frac{d}{dx}(\sec x \cdot y) = 2 \sin x$$

Integrating,

$$\int d/dx(\sec x \cdot y) = 2\int \sin x \, dx$$

$$y \cdot \sec x = -2\cos x + c$$

$$y = -\frac{2\cos x}{\sec x} + \frac{c}{\sec x}$$

$$\boxed{y = -2\cos^2 x + c\cos x}$$ General Solution.

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— More practice Questions:—

i) $x^2 \, dy/dx + 3xy = 1/x$, $y(1) = 1$

ii) $dy/dx + 2y = 4\cos 2x$, $y(\pi/4) = 2$

iii) $dy/dx \cos^2 x + 3y = 1$, $y(\pi/4) = 4/3$

iv) $dy/dx + y/x^2 = 2xe^{1/x}$, $y(0) = 1$

v) $\tan x \, dy/dx = 2y - 8$, $y(\pi/8) = 0$

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