31/7/13

(Lec no 17,18)

Non-Linear D.E

$$y' + P_{(x)}y = g_{(x)}y^a$$ {Homogeneous Linear D.E}

→ Bernouli's Equation:-

Sol:- Let $u = y^{1-a}$

Differentiate w.r.t x

$$u'_{(x)} = (1-a)y^{1-a-1}\,y'$$

$$u' = (1-a)y^{-a}y'$$

$$\therefore \quad y' = gy^a - Py$$

So,

$$\cancel{u' = (1-a)g - (1-a)P}$$

$$u' = (1-a)y^{-a}(gy^a - Py)$$

$$u' = (1-a)[gy^a - Py^{1-a}]$$

$$u' = (1-a)g - (1-a)Pu \qquad \because u = y^{1-a}$$

$$u' + ((1-a)P)u = (1-a)g_{(x)}$$

This is Linear D.E

— Practice Questions: —

Ques 1:- $y' = Ay - By^2$

Let $u = y^{1-2} = y^{-1}$

Diff w.r.t x

$$u' = -1y^{-2}\, y'$$

$$u' = -1y^{-2}(Ay - By^2) \quad \because y' = Ay - By^2$$

$$u' = -Ay^{-1} + By^0$$

$$u' = -A(u) + B \quad \because u = y^{-1}$$

$$\boxed{u' + Au = B}$$ Linear D.E

Integrating factors $= e^{\int P_x dx}$

$= e^{\int A dx}$

$= e^{A\int dx}$

$= e^{Ax}$

Multiply I.F with Linear D.E,

$$e^{Ax}u' + Ae^{Ax}u = Be^{Ax}$$

Integrate it,

$$\int \frac{d}{dx}(e^{Ax}u)\,dx = \int Be^{Ax}\,dx$$

$$ue^{Ax} = \frac{Be^{Ax}}{A} + C$$

$$u_{(x)} = B/A + ce^{-Ax}$$

$$y^{-1} = B/A + ce^{-Ax}$$

$$y = \frac{A}{B + ce^{-Ax}}$$

Ques 2:- $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3} \rightarrow y' + \left(\frac{1}{2x}\right)y = xy^{-3}$

Let $u = y^{1-(-3)} = y^4$

$$u' = 4y^3 y'$$

$$u' = 4y^3\left(xy^{-3} - \frac{1}{2x}y\right) \quad \therefore y' = xy^{3} - \frac{1}{2x}y$$

$$u' = 4x - \frac{2}{x}y^4$$

$$u' = 4x - 2/x\, u \quad \therefore u = y^4$$

$$\boxed{u' + \frac{2}{x}u = 4x}$$ Linear D.E

Integrating factor,

$$= e^{\int \frac{2}{x} dx}$$

$$= e^{2\int \frac{dx}{x}}$$

$$= e^{2 \ln x}$$

$$= x^2$$

Multiply I.F with x^2,

$$x^2 u' + \left(\frac{2}{x}\right) x^2 u = 4x(x^2)$$

$$\int \frac{d}{dx}(x^2 u)\, dx = \int 4x^3\, dx$$

$$x^2 u = \frac{4x^4}{4} + c$$

$$u = \frac{x^4}{x^2} + \frac{c}{x^2}$$

$$u = x^2 + cx^{-2}$$

$$\boxed{y^4 = x^2 + cx^{-2}}$$

Question no 3.- $dy/dx + 3/x\, y = x^2 y^2$

$$y' + \left(\frac{3}{x}\right) y = x^2 y^2$$

Let $u = y^{1-2} = y^{-1}$

$$u' = -1 y^{-2} y'$$

$$u' = -y^{-2}\left(x^2y^2 - (3/x) y\right)$$

$$u' = -x^2 + 3/x\, y^{-1}$$

$$u' = -x^2 + 3/x\, u \quad \because y^{-1} = u$$

$$\boxed{u' - (3/x) u = -x^2} \quad \text{L.D.E}$$

Integrating factor,

$$= e^{\int P_0 dx}$$

$$= e^{-3\int dx/x}$$

$$= e^{-3 \ln x}$$

$$= x^{-3}$$

Multiply I.F with L.D.E

$$x^{-3}u' - 3/x\,(x^{-3})(u) = -x^2(x^{-3})$$

Integrate it,

$$\int \frac{d}{dx}(x^{-3} \cdot u)\, dx = \int -\frac{1}{x}\, dx$$

$$x^{3} u = -\ln x + C$$

$$u = -\ln x (x^{3}) + C x^{3}$$

$$\frac{1}{y} = -x^{3} \ln x + C x^{3}$$

$$y = x^{3}[\ln x + C]$$