7/8/15

(Lecno 19, 20)

Second Order D.E

Second Order D.E is called Linear if,

$$y'' + P(x)y' + Q(x)y = r(x)$$

(Non-Homogeneous)

$$y'' + P(x)y' + Q(x)y = 0$$

(Homogeneous)

Solution of Homogeneous 2nd Order L.D.E :—

$$ay'' + by' + cy = 0 \quad \text{—— (i)}$$

Step 1- Let $y = e^{rx}$ —— (ii), is a solution.

Step 2- We will put value of y in eq (i)

$$a(e^{rx})'' + b(e^{rx})' + c(e^{rx}) = 0$$

$$ar^2e^{rx} + bre^{rx} + ce^{rx} = 0$$

$$e^{rx}(ar^2 + br + c) = 0$$

Since e^{rx} is a solution, e^{rx} can never be zero

So. $ar^2 + br + c = 0$ —— (iii)

eq (iii) is a characteristic equation of eq (i)

Since eq (iii) is quadratic in nature,

We will use quadratic formula,

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Case 1:-

If $b^2 > 4ac$ then $b^2 - 4ac > 0$
So roots will real and different
i.e

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

—: Practice Question for Case 1 :—

Qus 1:- $y'' + 2y' - y = 0$ ⟶ (ii) $y'(0) = -1, y(0) = 0$

Let, $y = e^{\delta x}$ — (iii)

put eq (iii) in eq (ii)

$$(e^{rx})'' + 2(e^{\delta x})' - (e^{rx}) = 0$$

$$r^2 e^{rx} + 2re^{rx} - e^{\delta x} = 0$$

$$e^{rx}[r^2 + 2r - 1] = 0$$

Characteristic Equation = $r^2 + 2r - 1$ —— (iii)

Using Quadratic formula,

$$r_{1,2} = \frac{-2 \pm \sqrt{4-4(1)(-1)}}{2(1)}$$

$$r_{1,2} = \frac{-2 \pm \sqrt{8}}{2} \Rightarrow \frac{-2 \pm 2\sqrt{2}}{2}$$

$$r_{1,2} = -1 \pm \sqrt{2}$$

$$r_1 = -1+\sqrt{2}, \quad r_2 = -1-\sqrt{2}$$

$$y = c_1 e^{(-1+\sqrt{2})x} + c_2 e^{(-1-\sqrt{2})x}$$

for initial Condition :-

$$y = c_1 e^{(-1+\sqrt{2})x} + c_2 e^{(-1-\sqrt{2})x} \quad \text{---(iv)}$$

$$y_{(0)} = 0$$

So, $x = 0$, $y = 0$

$$0 = c_1 e^0 + c_2 e^0$$

$$c_1 + c_2 = 0 \quad \text{---(v)}$$

$$c_1 = -c_2$$

Now differentiate eq iv w.r.t x,

$$y' = c_1(-1+\sqrt{2})e^{(-1+\sqrt{2})x} + c_2(-1-\sqrt{2})e^{(-1-\sqrt{2})x}$$

$y'_{(0)} = -1$

$x = 0 \quad y' = -1$

$-1 = c_1(-1+\sqrt{2}) + c_2(-1-\sqrt{2}) \quad \text{——— (ii)}$

put $c_1 = -c_2$

$-1 = -c_2(-1+\sqrt{2}) + c_2(-1-\sqrt{2})$

$-1 = c_2\left(-1(-1+\sqrt{2}) + (-1-\sqrt{2})\right)$

$-1 = c_2\left(1-\sqrt{2}-1-\sqrt{2}\right)$

$-1 = c_2(-2\sqrt{2})$

$c_2 = \frac{1}{2\sqrt{2}} \quad$ so $\quad c_1 = -\frac{1}{2\sqrt{2}}$