Differential Equation

03/4/15

(Lecture 1.2)

As we know that,

$$f = ma$$

$$= m\frac{dv}{dt} \quad \because a = dv/dt$$

$$= m\frac{d}{dt}\frac{dx}{dt} \quad \because v = dx/dt$$

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(Differential Equation Operation)

So, $$\boxed{F = m\frac{d^2x}{dt^2}}$$

Syllabus for Semester

Pre-mid

1st Order Equations
- DE types
- Solution of D.E
- Seperable
- Exact
- Non-exact
- Linear D.E
- Modeling Problems

Higher Order Equations
- Non-Homogenous D.E
- If Method
- Bernoulli Equation

Post-mid

- Modeling Problems
- Mass Spear Systems
- LR Circuits
- Solution of Non-Homogenous Higher Order D.E
- Variation of Parameter
- Undetermined coffiant
- System of Linear D.E
- Mango Problem

Equation:-

An equation is relationship b/w dependent & Independent variables.

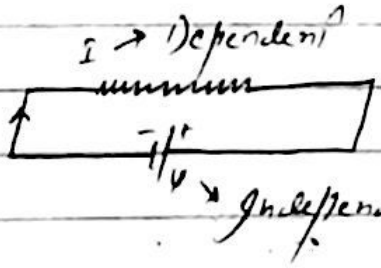
Types:-

It has following two types:-

i) Independent Variables

ii) Dependent Variables

i.e Ohm's Law



$\frac{d}{dV}(I)$ → Dependent

dV → Independent

Equation of Line:-

$$y = mx + c$$

- One variable should be Independent.

Differential Equation:-

An equation involving dependent & Independent variable and the derivative of dependent variable is called Differential Equation.

i.e Dependent $\frac{dy}{dx} + y = 0$

Independent

Order of Differential Equation:-

The order of differential equation is the order of the highest derivative that occur.

i.e.

i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + x = 0$ — 2nd Order D.E

ii) $y = 0$ — 0 order D.E

Degree of Differential Equation:-

The degree of differential equation is the power of the highest order derivative

i.e.

$$\left[\left(\frac{d^2y}{dx^2}\right)^{2/3}\right]^5 = \left(4x + dy/dx\right)^5$$

Degree = 2

Order = 2

Types of Differential Equations:-

i) Ordinary D.E

ii) Partial D.E

Ordinary Differential Equation:-

An equation involving only derivatives of one or more dependent variables w.r.t a single independent variables.

i.e $dy/dx + d^2/dx = 3$ (Ordinary D.E)

Partial Differential Equation:-

An equation involving partial derivatives of one or more dependent variables w.r.t one or more independent variables.

i.e. $$\frac{d^2u}{dx^2} + \frac{d^2v}{dy^2} + \frac{d^2w}{dz^2} = 0$$

How to solve differential Equation?

we will always solve the dependent variable by integrating equation, so that we can remove differential from equation.

Example :-

$$\frac{d^2x}{dt^2} = f/m$$

$$\int \frac{d}{dt}\left(\frac{dx}{dt}\right) dt = \int f/m \, dt$$

$$\int d\left(\frac{dx}{dt}\right) = f/m \int dt$$

$$x' = \frac{dx}{dt} = \left(f/m\right) t + c_1 \quad \text{---- (i)}$$

Again Integrate (i),

$$\int \frac{dx}{dt} dt = f/m \int t \, dt + \int c_1 \, dt$$

$$\boxed{x_{(t)} = \left(f/m\right)\left(\frac{t^2}{2}\right) + c_1 t + c_2}$$ General Solution

if $c_1 = 1$, $c_2 = 0$

$$\boxed{x_{(t)} = \frac{f}{2m} t^2 + t}$$ particular Solution after conditions