1/2/16

(Lecture 16) Sequence & Series

1, 2, 3, 4, 5, ... n

2, 4, 6, 8, 10, ... [nth term (2n)]

4, 9, 16, 25, ...

-: Root test and Ratio Tests :-

In case of $a_1 + a_2 + a_3 + a_4 + a_5 + \cdots$

$$\sum_1^{\infty} a_n$$

Let $\sum_1^{\infty} a_n$ be the series

Suppose

$$\lim_{n\to\infty} \sum_1^{\infty} a_n = l \quad , \text{ so}$$

i) If $l < 1$ (Series Convergent)

ii) If $l > 1$ (Series Divergent)

iii) If $l = 1$ (Test fail)

$$\lim_{x\to\infty} \frac{a_{n+1}}{a_n} = l$$

- Factorials

$\rightarrow \quad \frac{10!}{5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}}$

$\rightarrow \quad \frac{n!}{(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$

$\rightarrow \quad \frac{n^{n+1}}{n^n} = \frac{\cancel{n^n} \cdot n}{\cancel{n^n}} = n$

Scanned by CamScanner

Questions of Ratio Test

Question:- $\sum_{0}^{\infty} \frac{2^n}{(2n)!}$ (a_nth Term)

Sol:-

$$a_n = \frac{2^n}{(2n)!} \Rightarrow a_{n+1} = \frac{2^{n+1}}{2(n+1)!}$$

$$a_{n+1} = \frac{2^{n+1}}{(2n+2)!}$$

Now Apply Ratio Test

$$\lim_{n\to\infty} \frac{2^{n+1}}{(2n+2)!} \times \frac{(2n)!}{2^n}$$

$$\lim_{n\to\infty} \frac{2^n \cdot 2}{(2n+2)(2n+1)\cancel{(2n)!}} \times \frac{\cancel{(2n)!}}{\cancel{2^n}}$$

$$l = \lim_{n\to\infty} \frac{2}{(2n+2)(2n+1)}$$

$$l = 2/\infty = 0 \quad [\text{Series Convergent}]$$

Question 2:- $\sum_{1}^{\infty} \frac{n!}{n^n}$

$$= a_n = \frac{n!}{n^n}$$

$$a_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \times \frac{n^n}{n!}$$

$$= \lim_{n\to\infty} \frac{\cancel{(n+1)}\,\cancel{n!}}{(n+1)^n \cdot \cancel{(n+1)}} \times \frac{n^n}{\cancel{n!}}$$

$$= \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$$

$$= \lim_{n\to\infty} \left(\frac{n}{\cancel{n}(1+1/n)}\right)^n = \lim_{n\to\infty} \left(\frac{1}{1+1/n}\right)^n$$

$= 1/e < 1$ (Series Convergent)

Question 3:- $\sum_{1}^{\infty} \frac{7^n}{n(5^{n+1})}$

$$a_n = \frac{7^n}{n(5^{(n+1)})} \Rightarrow a_{n+1} = \frac{7^{(n+1)}}{(n+1)(5^{(n+2)})}$$

$$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{7^{n+1}}{(n+1)(5^{n+2})} \times \frac{n(5^{n+1})}{7^n}$$

$$l = \lim_{n\to\infty} \frac{\cancel{7^n} \cdot 7}{(n+1)\,\cancel{5^{n+1}} \cdot 5} \times \frac{n\ \cancel{5^{n+1}}}{\cancel{7^n}}$$

$$l = 7/5 \lim_{n\to\infty} \frac{n}{n+1} \Rightarrow 7/5 \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}(1+1/n)}$$

$$l = 7/5 \times 1$$

= Series Divergent

Question 4:-

$$\sum_{1}^{\infty} \frac{n}{n^2+1}$$

$$a_n = n/n^2+1 \Rightarrow a_{n+1} = \frac{n+1}{(n+1)^2+1}$$

$$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n+1}{(n+1)^2+1} \times \frac{n^2+1}{n}$$

$$l = \lim_{n\to\infty} \frac{n+1}{n^2+2n+1+1} \times \frac{n^2+1}{n}$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)\left(\frac{n^2+1}{n^2+2n+2}\right)$$

$$= \lim_{n\to\infty} \frac{\cancel{n}(1+1/n)}{\cancel{n}} \cdot \frac{\cancel{n^2}(1+1/n^2)}{\cancel{n^2}\left(1+\frac{2}{n}+\frac{2}{n^2}\right)}$$

$$= (1)(1/1) = 1$$

(Series Fail)