Subject   Date 7/03/15

(Lecture 23)

Triple Integrals

Ques:- $\int_0^x \int_0^y \int_0^z xyz \, dx \, dy \, dz$

Sol:-

$= \int_0^x \int_0^y yz \int_0^z x \, dx \, dy \, dz$

$= \int_0^x \int_0^y yz \left| \frac{z^2}{2} \right|_0^z dy \, dz$

$= \int_0^x \int_0^y yz \left(\frac{z^2}{2}\right) dy \, dz \Rightarrow \int_0^x \int_0^y y \frac{z^3}{2} \, dy \, dz$

$= \int_0^x \frac{z^3}{2} \int_0^y y \, dy \, dz \Rightarrow \int_0^x \frac{z^3}{2} \left| \frac{y^2}{2} \right|_0^y dz$

$= \int_0^x \frac{z^3}{2} \left(y^2/2 \right) dz \Rightarrow \int_0^x z^3 \left(y^2/4 \right) dz$

$= y^2/4 \left| \frac{z^4}{4} \right|_0^x \Rightarrow = \left(y^2/4 \right)\left(x^4/4 \right)$

$= \boxed{x^4 y^2 / 16}$ Ans

Ques $\int_0^x \int_0^y \int_0^z (x+y+z)\, dy\, dx\, dz$

Sol

$$= \int_0^x \int_0^y \left| xy + \frac{y^2}{2} + zy \right|_0^z dx\, dz$$

$$= \int_0^x \int_0^y \left(xz + \frac{z^2}{2} + z^2 \right) dx\, dz$$

$$= \int_0^x \int_0^y \left(xz + \frac{3z^2}{2} \right) dx\, dz \Rightarrow \int_0^x \left| \frac{x^2 z}{2} + \frac{3}{2} x z^2 \right|_0^y dz$$

$$= \int_0^x \left(\frac{y^2 z}{2} + \frac{3}{2} z^2 y \right) dz \Rightarrow \frac{1}{2} \int_0^x \left(y^2 z + 3z^2 y \right) dz$$

$$= \frac{1}{2} \left| y^2 \frac{z^2}{2} + z^3 y \right|_0^x$$

$$= \frac{1}{2} \left(y^2 \frac{x^2}{2} + x^3 y \right)$$

$$= \boxed{y^2 x^2/4 + x^3 y/2}$$

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Subject: ______ Date: ______

Ques:- $\int_1^2 \int_x^{4/x} \int_0^{xy} xy \, dz \, dy \, dx$

Sol:- $= \int_1^2 \int_x^{4/x} xy \int_0^{xy} dz \, dy \, dx$

$= \int_1^2 \int_x^{4/x} xy \, \left| z \right|_0^{xy} dy \, dx$

$= \int_1^2 \int_x^{y/x} xy \, (xy - 0) \, dy \, dx$

$= \int_1^2 \int_x^{y/x} x^2 y^2 \, dy \, dx \Rightarrow \int_1^2 x^2 \int_x^{y/x} y^2 \, dy \, dx$

$= \int_1^2 x^2 \left| y^3/3 \right|_x^{4/x} dx \Rightarrow \int_1^2 \frac{x^2}{3} \left(\frac{64}{x^3} - x^3 \right) dx$

$= \int_1^2 x^2/3 \left(64/x^3 - x^3 \right) dx \Rightarrow \int_1^2 \left(\frac{64}{3x} - \frac{x^5}{3x} \right) dx$

$= \left| \frac{64}{3x} \ln x - \frac{x^6}{18} \right|_1^2$

$= \left(\frac{64}{3} \ln 2 - \frac{2^6}{18} \right) - \left(64/3 \ln 1 - 1/18 \right)$

$= \left(6.42 - 256/18 \right) - \left(0 - 1/18 \right)$

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Ques $\int_0^1 \int_y^{2y} \int_0^x y e^z \, dz \, dx \, dy$

Sol. $= \int_0^1 \int_y^{2y} y \int_0^x e^z \, dz \, dx \, dy$

$= \int_0^1 \int_y^{2y} y \left| e^z \right|_0^x dx \, dy$

$= \int_0^1 \int_y^{2y} y(e^x - 1) dx \, dy$

$= \int_0^1 y \int_y^{2y} (e^x - 1) \, dx \, dy \Rightarrow \int_0^1 y \left| e^x - x \right|_y^{2y} dy$

$= \int_0^1 y \left[(e^{2y} - 2y) - (e^y - y) \right] dy \Rightarrow \int_0^1 y \left(e^{2y} - 2y - e^y - y \right) dy$

$\int_0^1 y \left(e^{2y} - y - e^y \right) dy \Rightarrow \int_0^1 \left(y e^{2y} - y^2 - y e^y \right) dy$

when we have different functions
That are multiplying with each other then we always
solves their integration by parts.

$= \int_0^1 y e^{2y} - y^3/3 - \int_0^1 y e^y \, dy$

$= y \left| \frac{e^{2y}}{2} \right|_0^1 - \int_0^1 \left| \frac{e^{2y}}{2} \right|_0^1 \cdot 1 \, dy - y^3/3 - y \left| \frac{e^y}{1} \right|_0^1 - \int \left| \frac{e^y}{1} \right|_0^1 \cdot 1 \, dy$

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Practice Questions for Triple Integrals

Ques:- $\int_0^4 \int_0^{\sqrt{25-x^2}} \int_{y-x}^{y+x} y \, dz \, dy \, dx$

Ans:- $\boxed{136}$

Ques:- $\int_1^2 \int_x^{y/x} \int_0^{2y} xy \, dz \, dy \, dx$

Ans:- $\frac{1}{6}(128 \ln - 21)$

Ques:- $\int_0^1 \int_y^{2y} \int_0^x ye^z \, dz \, dx \, dy$

(Important Question)

Ans:- $\frac{1}{12}(3e^2 - 13)$

Ques:- $\int_0^1 \int_0^1 \int_0^1 (u^2 + v^2 + w^2) \, dw \, dv \, du$

Ans:- 1

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