(Lecture 4)

Chain Rule

Formula:-

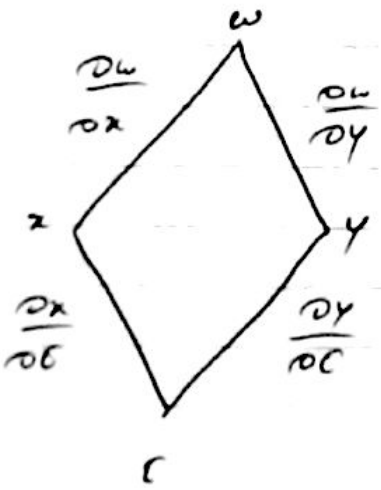
$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

Formula for two variables.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

Formula for three variables

We can also drive this formula by using this kite rule



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Practice Questions for Chain Rule

Qno 1:- Find $\frac{\partial w}{\partial t}$ if $w = xy$, $x = \cos t$, $y = \sin t$

Sol:- $$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{\partial (xy)}{\partial x}\frac{\partial (\cos t)}{\partial t} + \frac{\partial (xy)}{\partial y}\frac{\partial (\sin t)}{\partial t}$$

$$= -y \sin t + x \cos t$$

$$\boxed{\frac{\partial w}{\partial t} = x \cos t - y \sin t}$$

Qno 2:-

Find $\frac{\partial w}{\partial t}$ if $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$ at $t = 0$

Sol:- $$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$= \frac{\partial (xy+z)}{\partial x}\frac{\partial (\cos t)}{\partial t} + \frac{\partial (xy+z)}{\partial y}\frac{\partial (\sin t)}{\partial t} + \frac{\partial (xy+z)}{\partial z}\frac{\partial (t)}{\partial t}$$

$$= -y \sin t + x \cos t + 1$$

at $t = 0$

$$= -y \sin(0) + x \cos(0) + 1$$

$$= x1 + 1 \cdot 1$$

$$\boxed{\partial w / \partial t = x + 1}$$

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Qno3:- Find $\frac{\partial w}{\partial t}$ if $w = x^2+y^2$, $x = \cos t$, $y = \sin t$ and $t = t$

Sol:-

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{\partial(x^2+y^2)}{\partial x}\frac{\partial(\cos t)}{\partial t} + \frac{\partial(x^2+y^2)}{\partial y}\frac{\partial(\sin t)}{\partial t}$$

$$= -2x\sin t + 2y\cos t$$

at $t = \pi$

$$\frac{\partial w}{\partial t} = -2x\sin(\pi) + 2y\cos(\pi)$$

$$\boxed{\frac{\partial w}{\partial t} = 2y}$$

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