

(Lecture  
5)

## Chain Rule

Question:- Find  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial s}$ , if

i)  $w = x + 2y + z^2$ ,  $x = r/s$ ,  $y = r^2 + \ln s$ ,  $z = 2r$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial r}$$

$$= \frac{\partial(x+2y+z^2)}{\partial x} \frac{\partial}{\partial r} \left( \frac{r}{s} \right) + \frac{\partial(x+2y+z^2)}{\partial y} \frac{\partial(r^2 + \ln s)}{\partial r} + \frac{\partial(x+2y+z^2)}{\partial z} \frac{\partial(2r)}{\partial r}$$

$$= 1 \cdot \frac{1}{s} + 2 \cdot 2r + 2z \cdot 2$$

$$= \frac{1}{s} + 4r + 4z$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial s}$$

$$= \frac{\partial(x+2y+z^2)}{\partial x} \frac{\partial}{\partial s} \left( \frac{r}{s} \right) + \frac{\partial(x+2y+z^2)}{\partial y} \frac{\partial(r^2 + \ln s)}{\partial s} + \frac{\partial(x+2y+z^2)}{\partial z} \frac{\partial(2r)}{\partial s}$$

$$= 1 \cdot \left( -\frac{1}{s^2} \right) + 2 \cdot \left( \frac{1}{s} \right) + 2z \cdot 0$$

$$= -\frac{1}{s^2} + \frac{2}{s} + 0$$

Question:- find  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial s}$  if  $w = x^2 + y^2$ ,  $x = r - s$   
 $y = r + s$

(Important Question) Soln:-

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r}$$

$$\frac{\partial w}{\partial r} = \frac{\partial (x^2 + y^2)}{\partial x} \frac{\partial (r - s)}{\partial r} + \frac{\partial (x^2 + y^2)}{\partial y} \frac{\partial (r + s)}{\partial r}$$

$$\frac{\partial w}{\partial r} = 2x \cdot 1 + 2y \cdot 1$$

$$\boxed{\frac{\partial w}{\partial r} = 2x + 2y}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s}$$

$$= \frac{\partial (x^2 + y^2)}{\partial x} \frac{\partial (r - s)}{\partial s} + \frac{\partial (x^2 + y^2)}{\partial y} \frac{\partial (r + s)}{\partial s}$$

$$= 2x \cdot -1 + 2y \cdot 1$$

$$\boxed{\frac{\partial w}{\partial s} = -2x + 2y}$$