

(Lecture
8)

Double Integrals

-: Practice of Simple Integrals:-

$$\begin{aligned} \text{i) } \int x dy \\ &= x \int dy \\ &= \boxed{xy} \text{ Ans} \end{aligned}$$

$$\text{ii) } \int_a^b x dx$$

$$\begin{aligned} &= \left| \frac{x^2}{2} \right|_a^b \\ &= \left(\frac{b^2}{2} \right) - \left(\frac{a^2}{2} \right) \end{aligned}$$

$$\text{iii) } \int_0^1 x y dy$$

$$= x \int_0^1 y dy$$

$$= x \left| \frac{y^2}{2} \right|_0^1 = x \left(\frac{1}{2} - \frac{0}{2} \right)$$

-: Double Integrals:-

(Practice Questions)

Question no 1:-

$$g = \int_0^1 \int_0^2 xy dx dy$$

w.r.t x

$$= \int_0^1 y \int_0^2 x dx dy$$

$$= \int_0^1 y \left(\frac{x^2}{2} \right)_0^2 dy$$

$$= \int_0^1 y (2) dy$$

$$= \int_0^1 2y dy$$

$$= 2 \int_0^1 y dy$$

$$= 2 \left| \frac{y^2}{2} \right|_0^1$$

$$= 2 \left(\frac{1}{2} \right) = 1$$

$$\boxed{g = 1}$$

(Important Question)



Question 2:-

$$\begin{aligned}
 I &= \int_0^1 \int_0^2 (x^2 + y^2) dy dx \\
 &= \int_0^1 \left[x^2 y + \frac{y^3}{3} \right]_0^2 dx \\
 &= \int_0^1 \left(x^2(2) + \frac{(2)^3}{3} - x^2(0) + \frac{(0)^3}{3} \right) dx \\
 &= \int_0^1 (2x^2 + 8/3 - x^2 - 1/3) dx \\
 &= \int_0^1 (x^2 + 7/3) dx \\
 &= \left[\frac{x^3}{3} + \frac{7}{3}x \right]_0^1 \\
 &= \left(\frac{(1)^3}{3} + \frac{7}{3} \right) - \left(\frac{0}{3} + \frac{7}{3}(0) \right) \\
 &= \left(\frac{1}{3} + \frac{7}{3} \right) = \frac{8}{3}
 \end{aligned}$$

$I = \frac{8}{3}$

Question 3:-

$$\begin{aligned}
 I &= \int_0^x \int_0^y dy dx \\
 &= \int_0^x \left[y \right]_0^y dx \\
 &= \int_0^x y dx \\
 &= y \int_0^x dx \\
 &= y \left[x \right]_0^x \\
 &= y(x-0) = yx
 \end{aligned}$$

$I = yx$