Lesson 1
Introduction to Path Planning
Graph Searches: BFS and DFS

DASL Summer Program

Path Planning

References: http://robotics.mem.drexel.edu/mhsieh/Courses/MEM380I/index.html
http://dasl.mem.drexel.edu/Hing/BFSDFSTutorial.htm
http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/16311/www/current/syllabus.html
http://www.cs.cmu.edu/~motionplanning/syllabus%202010a-2.html

Course Description

- 6-week course to introduce path planning and related algorithms
- Course consists of lectures with practical implementations using Matlab and the NXT Tribot Robot
- Course will culminate with a path planning challenge using a robot to navigate a course with obstacles

Syllabus

- Week 1 – BFS and DFS
- Week 2 – A* and Dijkstra's Algorithm
- Week 3 – Potential Fields
- Week 4 – Randomized Path Planning
- Week 5 – SLAM
- Week 6 – Path Planning Challenge

Homework

- One assignment per week
- Matlab implementation of planning technique or algorithm
- Should require ~2 hours to complete

Student Introductions

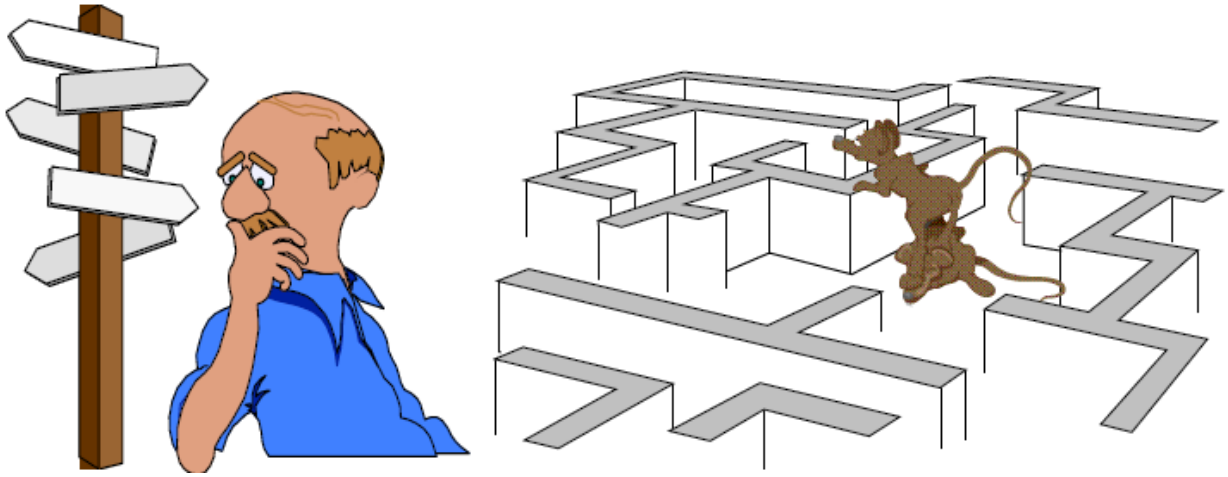
- Name
- Major, School, Year
- Hometown
- Hobbies
- Something interesting about you
- What you hope to get out of the course

Instructor

- Christopher Korpela
- cmk325@drexel.edu, christopher.korpela@gmail.com
- 1st year PhD Student in EE
- Bel Air, MD
- Soccer and running

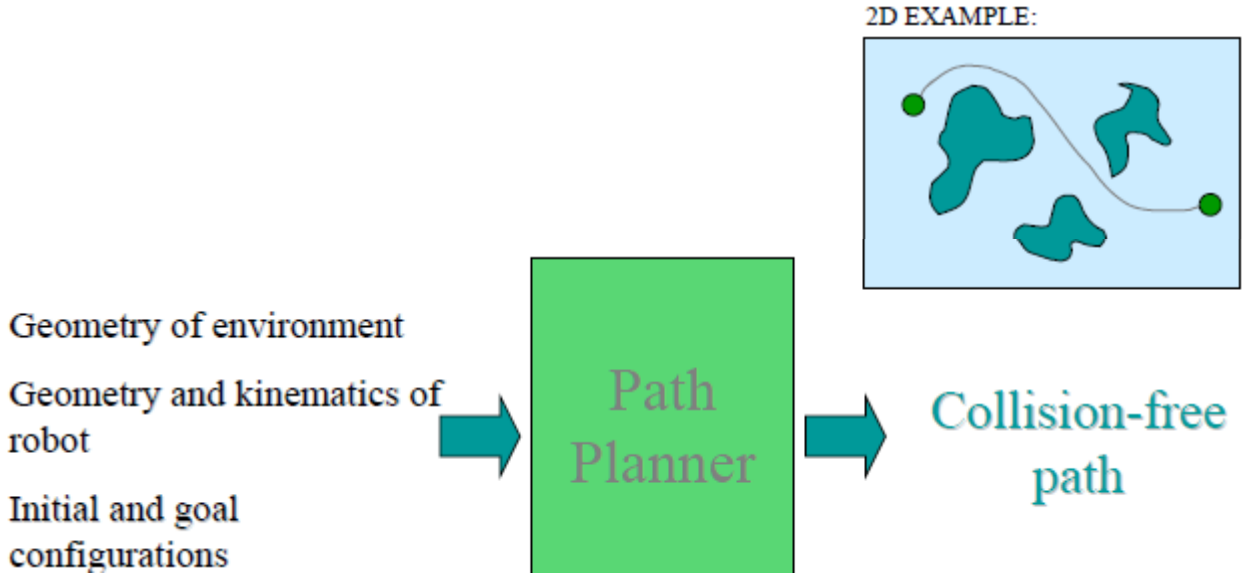
What is Path (Motion) Planning?

- Process of detailing a task into discrete motions
- Example: navigating a mobile robot from a start point to a distant end point



Basic Path Planning

- Problem Statement: Compute a continuous sequence of collision-free robot configurations connecting the initial and goal configurations



Live Motion Planning Experiments

- Person 1 walks through some obstacles
- Person 1, looking at Person 2, directs Person 2 through obstacles
- Person 1, looking at Person 2 with eyes closed, directs Person 2 through obstacles
- Person 1, looking at a map and not Person 2, whose eyes are still closed, directs Person 2 through obstacles
- Person 1, looking at an object and Person 2, whose eyes are closed, directs Person 2 to grab an obstacle
- How do we do the last experiment with a map?

What did we assume?

- Perfect sensors?
  - What information
  - Uncertainty
- Perfect control?
  - What controls?
  - Uncertainty
- Perfect thinking?
  - Knowledge of the world? Complete?
  - Processing the world? Everything?

Path Planning Considerations

- Planning tasks
  - Navigation
  - Coverage
  - Localization
  - Mapping
- Properties of the robot
  - degrees of freedom
  - holonomic or not
  - kinematic vs. dynamic
- Properties of Algorithms
  - Optimality
  - Computational complexity
  - Completeness
- Resolution completeness
- Probabilistic completeness
  - Online vs. offline
  - Sensor-based vs. not
  - Feedback or not

Basics: Metrics

- There are many different ways to measure a path:
  - Time
  - Distance traveled
  - Expense
  - Distance from obstacles
  - Etc…

Mathematical Rigor

Symbol	Meaning
$\exists$	there exists
$\forall$	for all
∞	infinity
$\in$	element
$\notin$	not in
s.t.	such that
$\mathbb{R}$	real numbers
$\mathbb{R}^m$	m-dimensioned real numbers
$\bigcup$	union
$\bigcap$	intersection
$\setminus$	set difference
$\Rightarrow$	implies. $p \rightarrow q$ is p implies q
$\Leftarrow$	implies. $q \rightarrow p$ is q implies p
$\Longleftrightarrow$	if and only if
S^1	a circle
∇	gradient
D	differential or distance to closest obstacle (depending on context)
d_i	distance to obstacle i in either the workspace or configuration space (depending on context)
$d(x, y)$	distance between the two points x and y
Null	null space

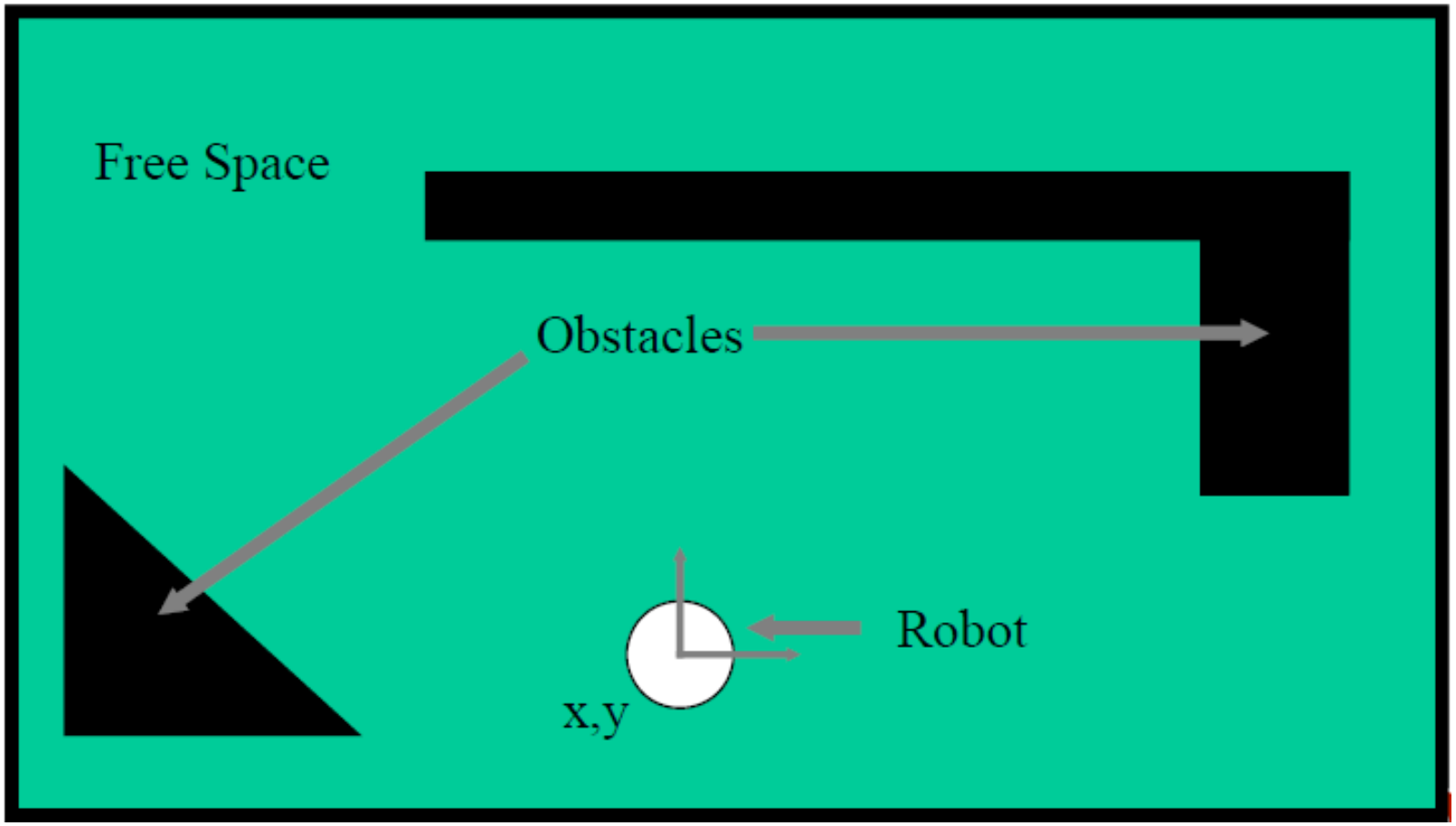
J	Jacobian
Γ	Christoffel symbol
RM	roadmap
$\mathcal{W}$	workspace
$\mathcal{Q}$	configuration space
$\mathcal{Q}_{\text{free}}$	free space
$x(k)$	state at time k
$\|\|x\|\|$	norm of x
$\subseteq$	subset of
$\subset$	strict subset of
cl(A)	closure of A
T^n	n-dimensional torus
S^n	n-dimensional sphere in $\mathbb{R}^{n+1}$
$SO(n)$	special orthogonal group
$SE(n)$	special Euclidean group
$B_\epsilon(q)$	open ball of radius ϵ centered at q
Df	differential of f
∇f	gradient of f
∇	affine connection
$\nabla_{Y_1} Y_2$	covariant derivative of Y_2 with respect to Y_1
C^0	continuous
C^n	n times differentiable
$\langle x, y \rangle$	inner product of x and y
$\mathcal{I}$	identity matrix
atan2(y, x)	returns angle to (x, y) in the plane in range $[-\pi, \pi)$
$T_x\mathcal{M}$	tangent space of $\mathcal{M}$ at x
$T\mathcal{M}$	tangent bundle of $\mathcal{M}$
$[f, g]$	Lie bracket of vector fields f, g
$\overline{\text{Lie}}(\mathcal{G})$	the Lie algebra of a set of vector fields $\mathcal{G}$
$\overline{\mathcal{D}}$	involutive closure of the distribution $\mathcal{D}$
$\mathcal{U}_\pm$	control set positively spanning $\mathbb{R}^m$
$\mathcal{U}_+$	control set spanning $\mathbb{R}^m$
$\langle Y_1 : Y_2 \rangle$	the symmetric product of vector fields Y_1 and Y_2

The World consists of

- Obstacles
  - Already occupied spaces of the world
  - In other words, robots can’t go there
- Free Space
  - Unoccupied space within the world
  - Robots “might” be able to go here
  - To determine where a robot can go, we need to discuss what a *Configuration Space* is

Robots

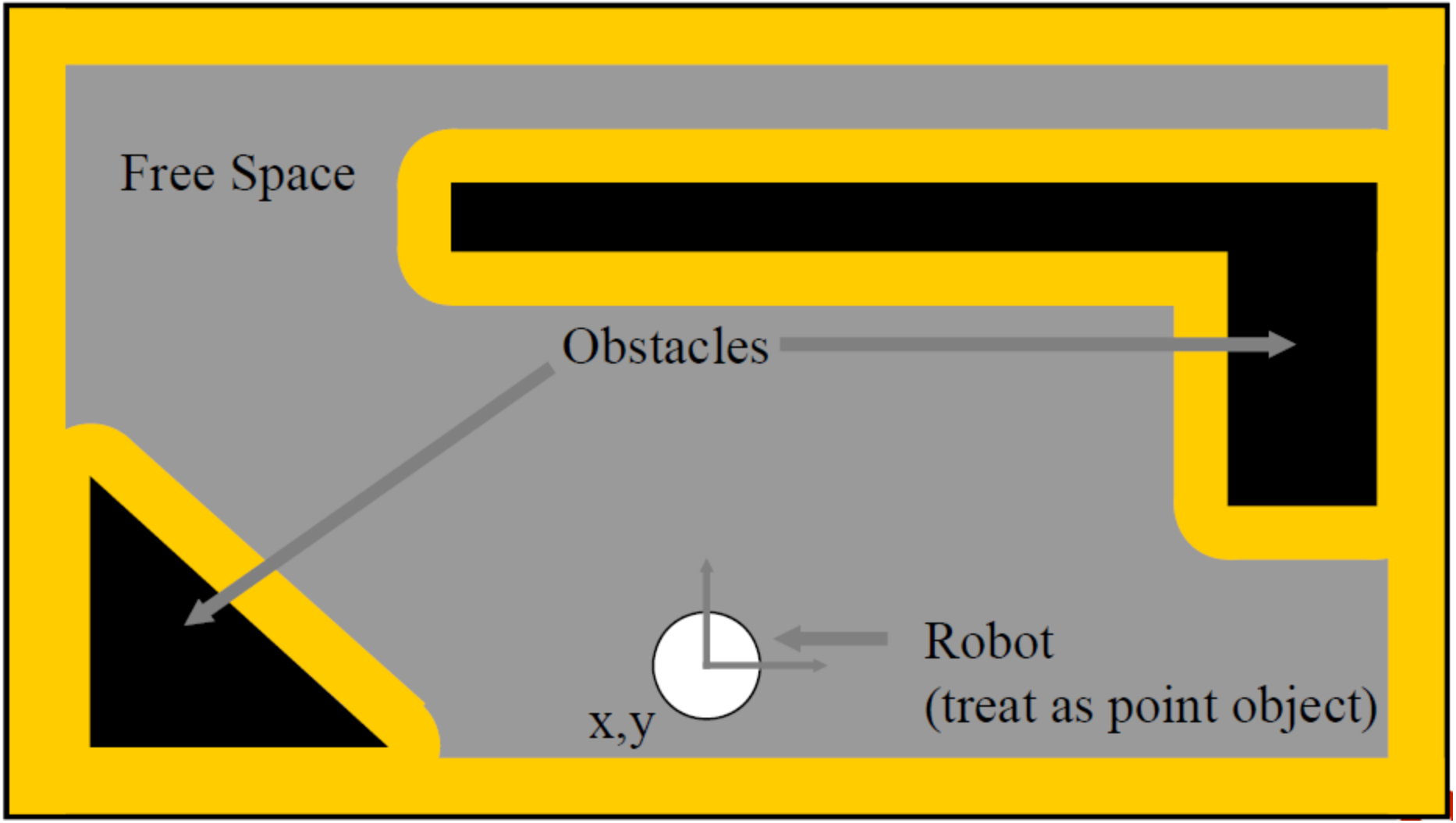
Robot and Environment



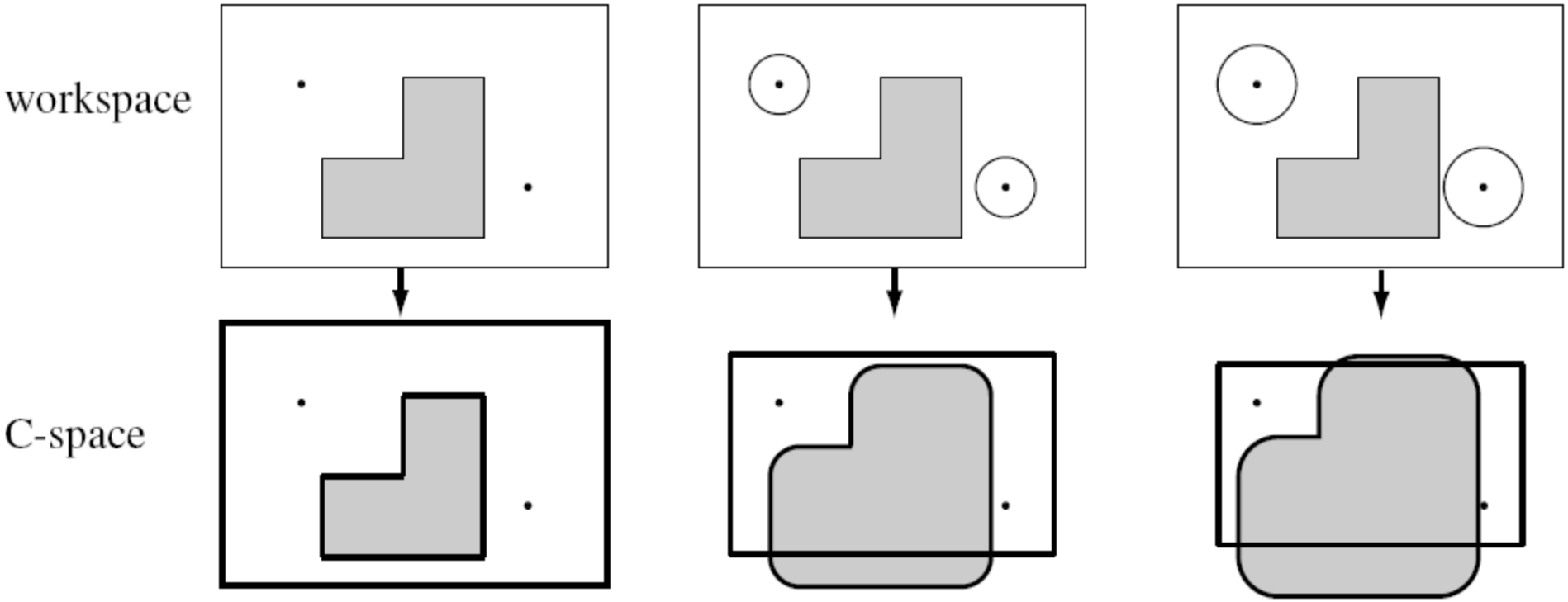
The Configuration Space

- What it is
  - A set of “reachable” areas constructed from knowledge of both the robot and the world
- How to create it
  - First abstract the robot as a point object. Then, enlarge the obstacles to account for the robot’s footprint and degrees of freedom
  - In our example, the robot was circular, so we simply enlarged our obstacles by the robot’s radius (*note the curved vertices*)

Configuration Space: Accommodate Robot Size



Trace Boundary of Workspace



Motion Planning Statement

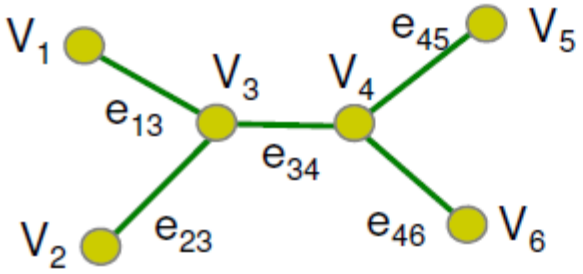
- If **W** denotes the robot's workspace,
- And $\mathbf{C_i}$ denotes the i'th obstacle,
- Then the robot's free space, **FS**, is defined as:

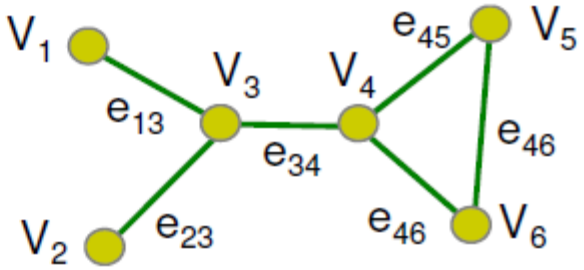
$$\mathbf{FS = W \setminus (\bigcup C_i)}$$

- And a path $\mathbf{c \in C^0}$ is $\mathbf{c : [0,1] \rightarrow FS}$
- where **c(0)** is $\mathbf{q_{start}}$ and **c(1)** is $\mathbf{q_{goal}}$

Motion Planning as Graph Searches

- Abstract representation of a *collection of items*
- A collection of vertices & edges

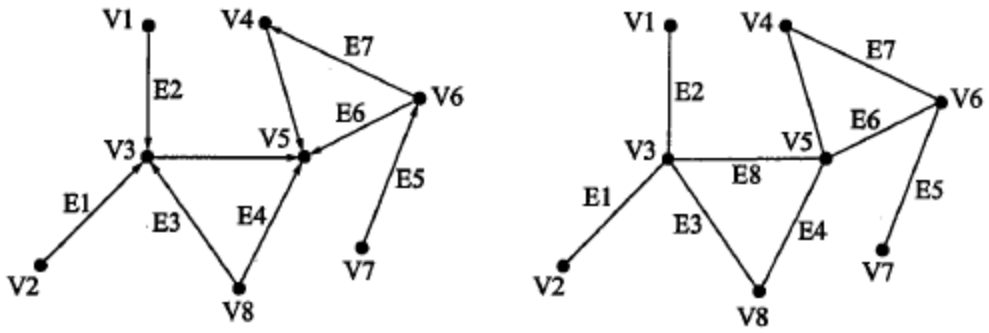




- Terminology
  - Adjacency
  - Paths and Cycles
  - Degree of a vertex
  - Fully connected graphs

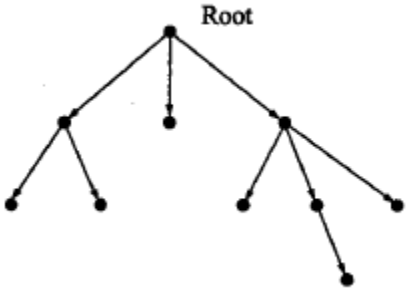
Types of Graphs

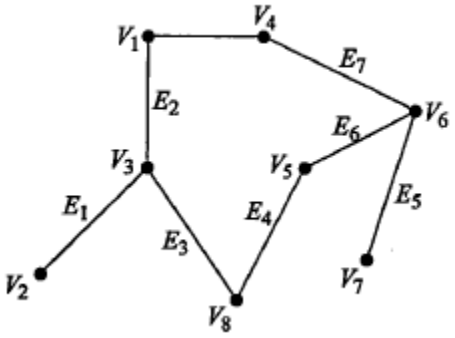
- Directed vs. Undirected
- Weighted vs. Unweighted



Types of Graphs

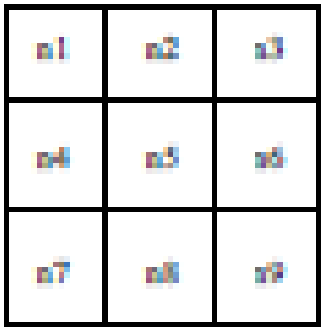
- Cyclic vs. Acyclic
- Trees





Grids


n1
n2
n3
n4
n5
n6
n7
n8
n9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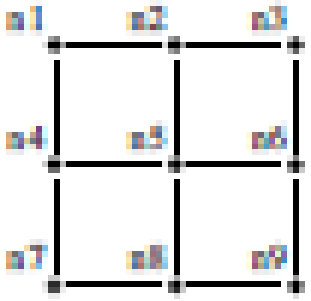
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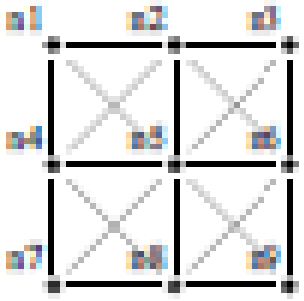
n1
n2
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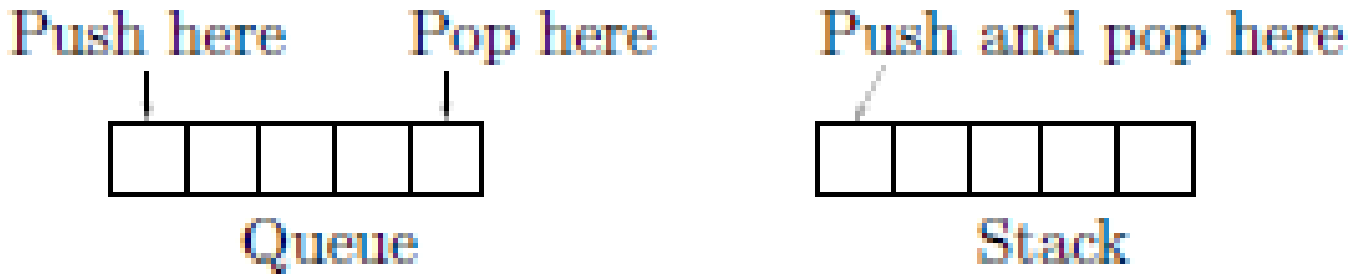
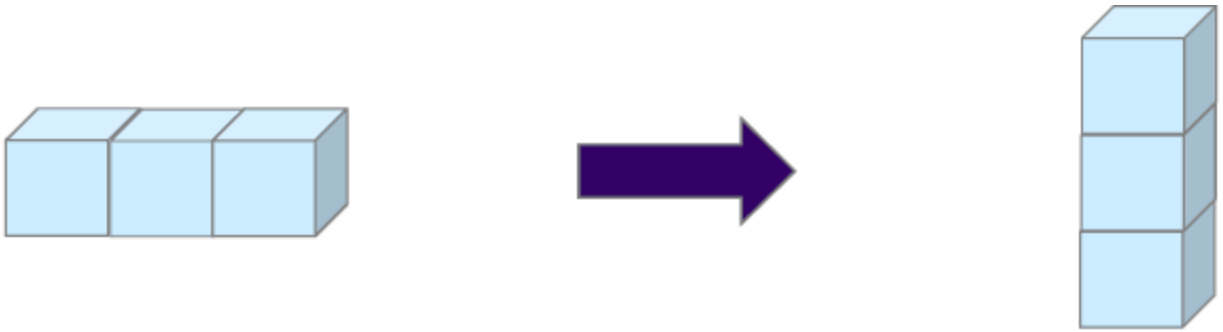
n9

Stacks and Queues

- Stack: First in, Last out (FILO)
- Queue: First in, First out (FIFO)



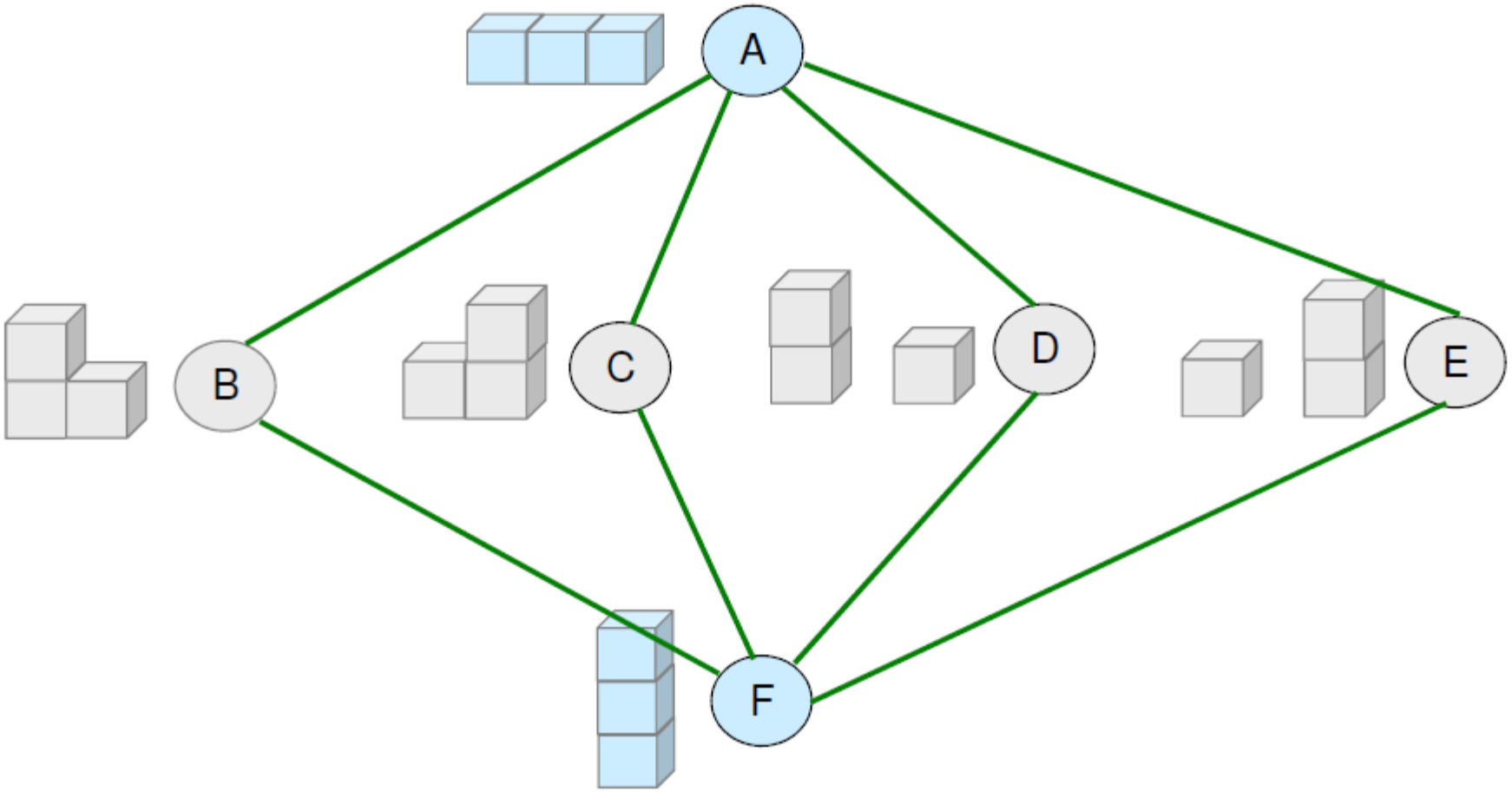
Graph Representation: Example



- Objective: We want to represent all possible ways to get from configuration A to B using a graph

HOW?

Graph Representation: Example



Search in Path Planning

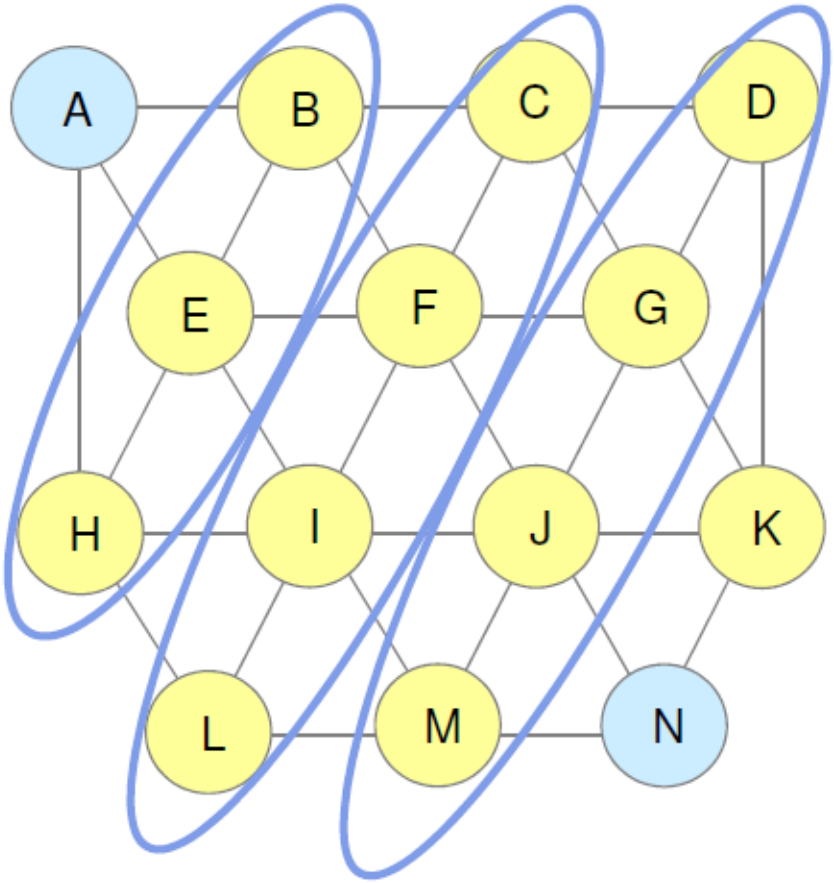
- Find a path between two locations in an *unknown, partially known, or known environment*
- Search Performance
  - Completeness
  - Optimality
  - Operating Costs
  - Space Complexity
  - Time Complexity

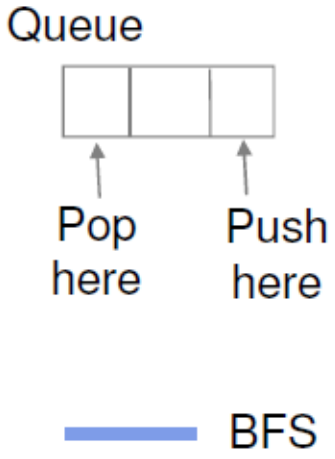
Search

- Uninformed Search
  - Use no information obtained from the environment
  - Blind search: BFS (Wavefront), DFS
- Informed Search
  - Use evaluation function
  - More efficient
  - Heuristic Searchers: A*, D*, etc.

Uniformed Search: BFS

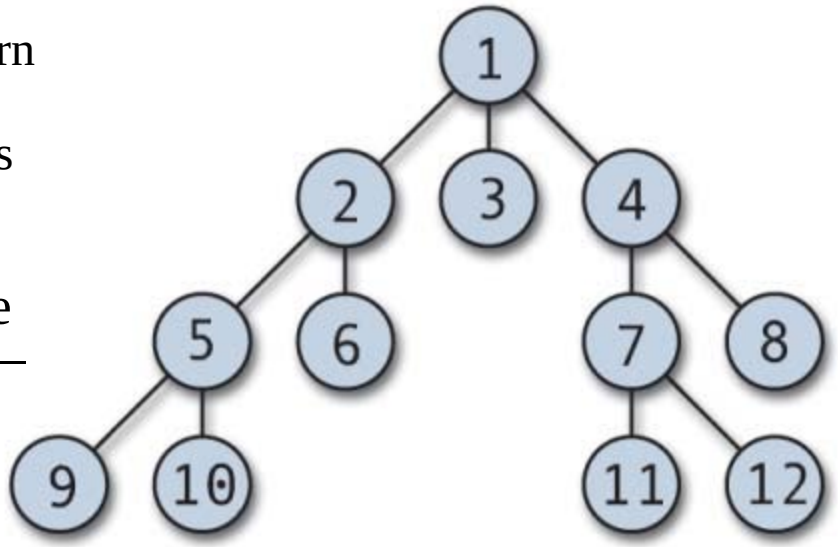
- Graph search from A to N





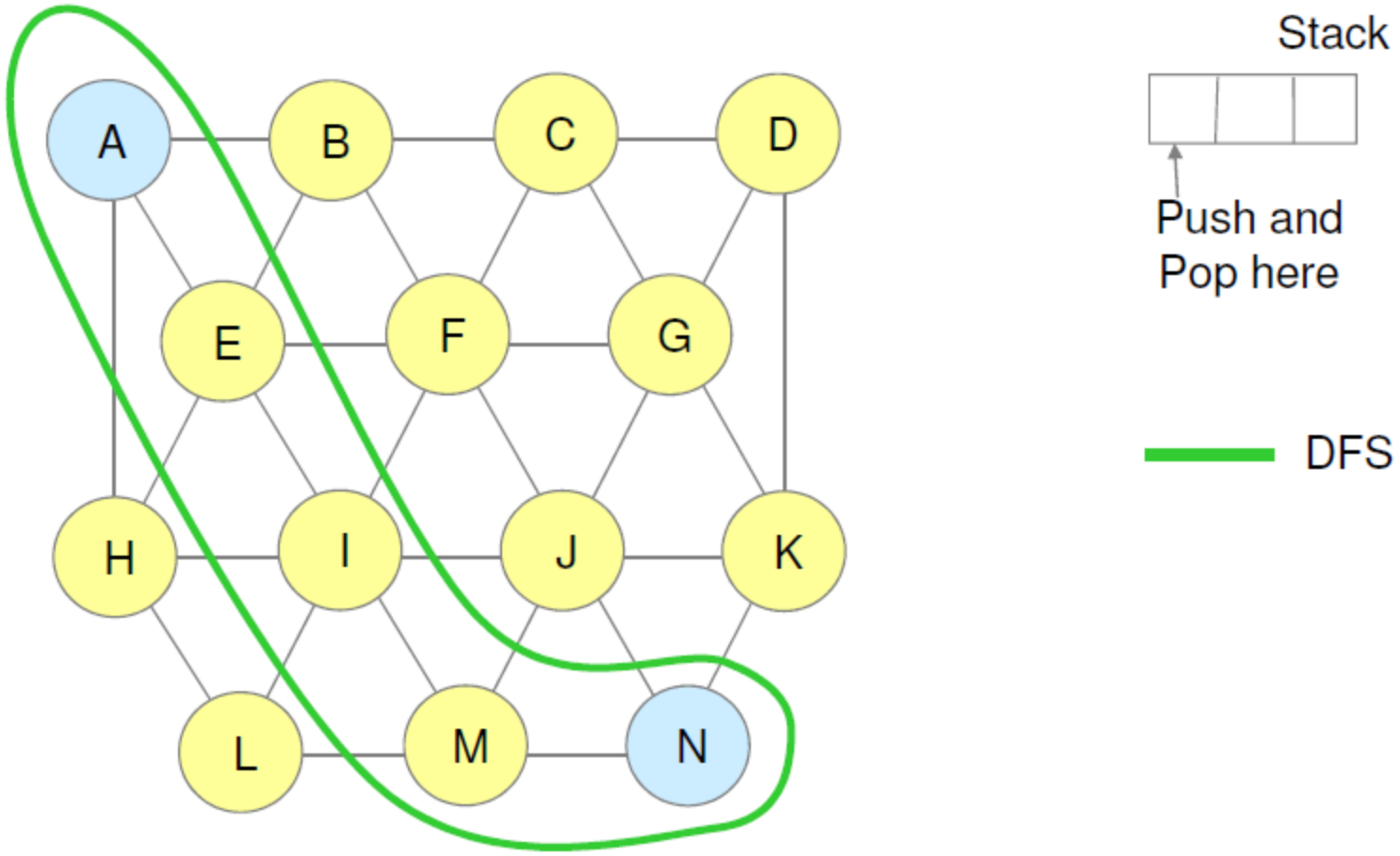
Breadth First Search

1. Enqueue the root node.
2. Dequeue a node and examine it.
   - If the element sought is found in this node, quit the search and return a result.
   - Otherwise enqueue any successors (the direct child nodes) that have not yet been discovered.
3. If the queue is empty, every node on the graph has been examined – quit the search and return "not found".
4. If the queue is not empty, repeat from Step 2.



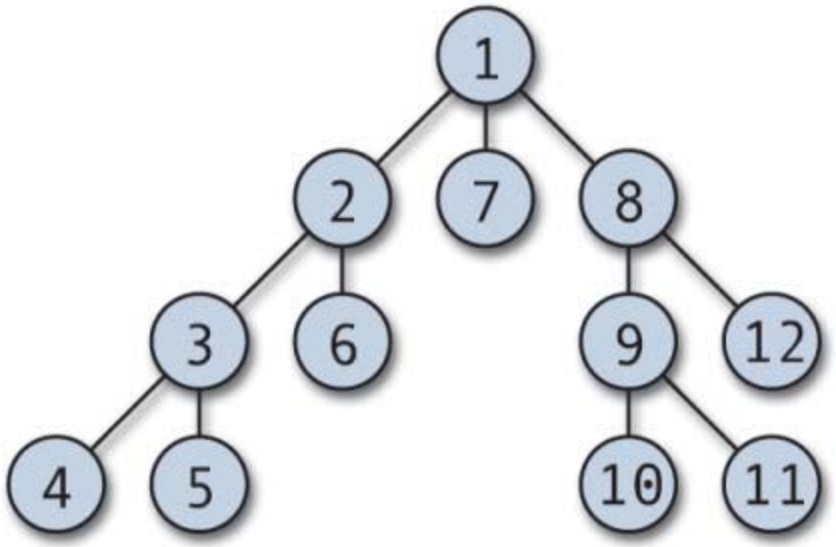
Uninformed Search: DFS

- Graph search from A to N

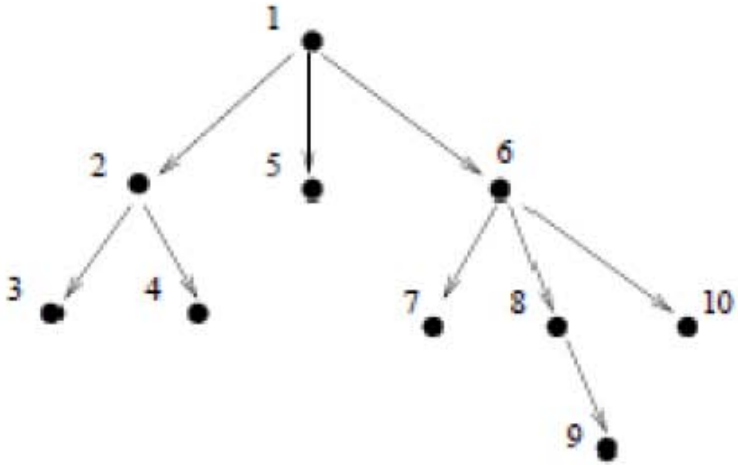


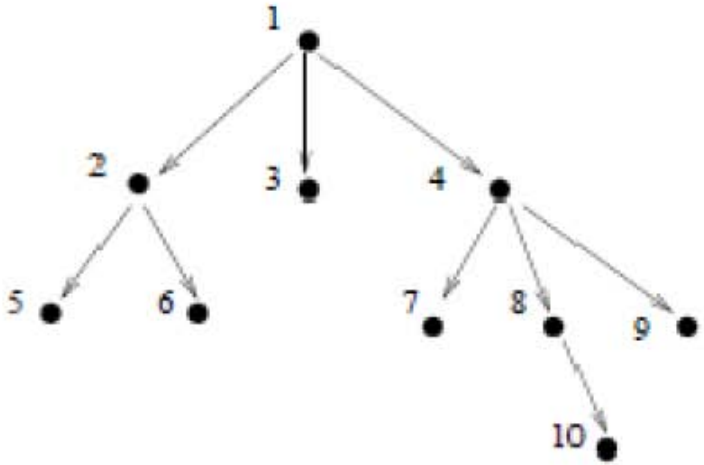
Depth First Search

```
algorithm dft(x)
   visit(x)
   FOR each y such that (x,y) is an edge
      IF y was not visited yet
         THEN dft(y)
```



Depth First and Breadth First Searches





DASL BFS-DFS Tutorial

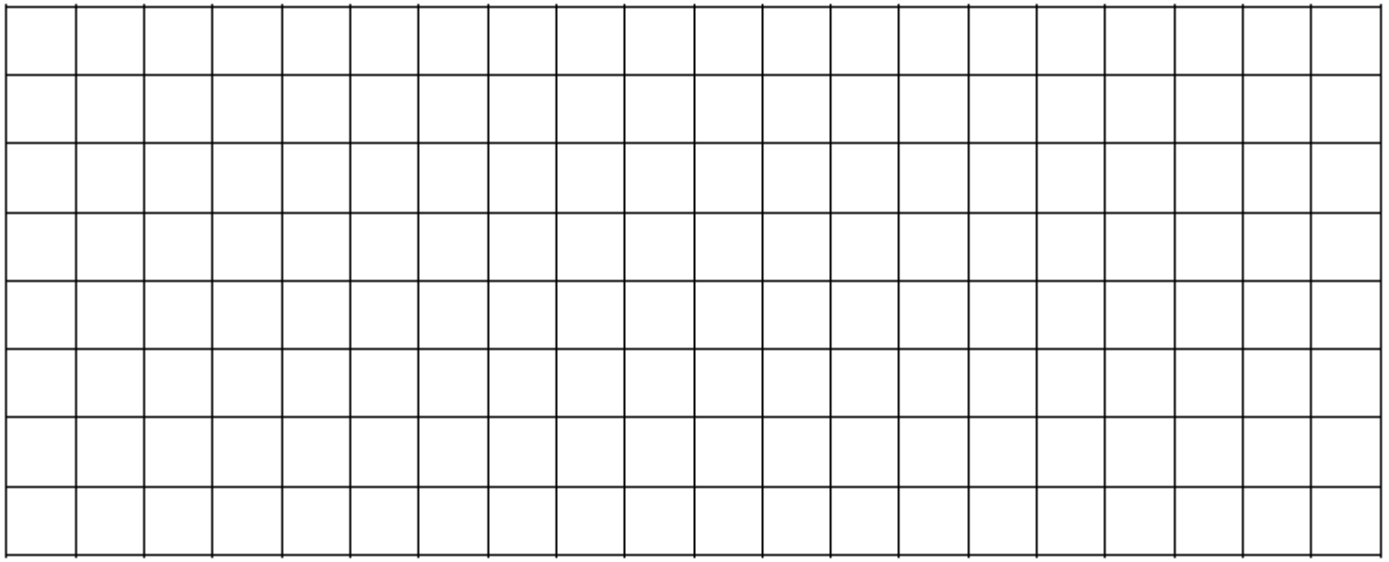
- http://dasl.mem.drexel.edu/Hing/BFSDFSTutorial.htm
- Download m-files
- Change obstacle configuration

The Wavefront Planner

- A common algorithm used to determine the shortest paths between two points
  - In essence, a breadth first search of a graph
- For simplification, we'll present the world as a two-dimensional grid
- Setup: Label free space with 0
- Label start as START
- Label the destination as 2

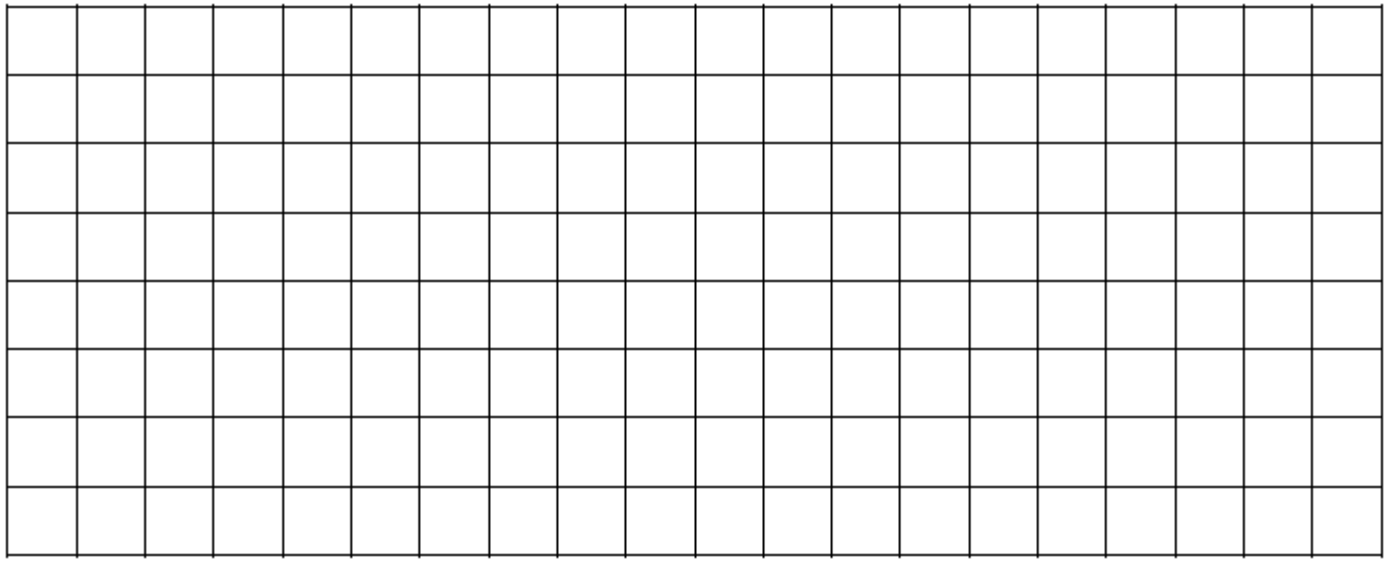
Representations

- World Representation
  - You could always use a large region and distances
  - However, a grid can be used for simplicity



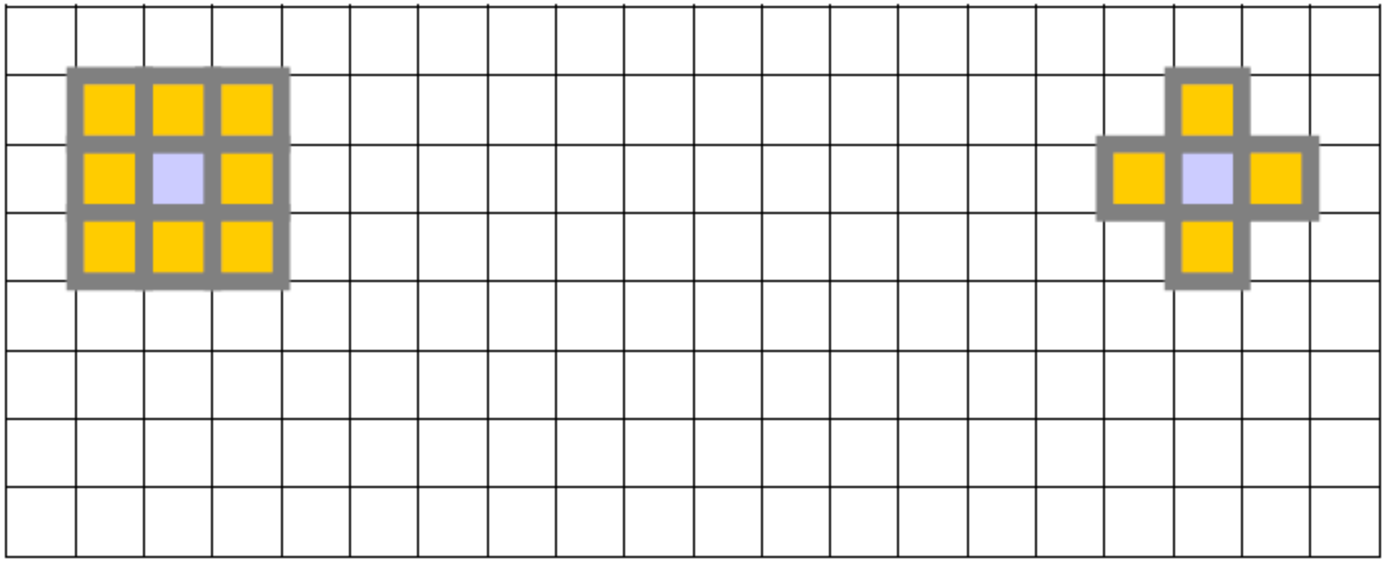
Representations: A Grid

- Distance is reduced to discrete steps
  - For simplicity, we'll assume distance is uniform
- Direction is now limited from one adjacent cell to another
  - Time to revisit Connectivity



Representations: Connectivity

- 8-Point Connectivity 4-Point Connectivity

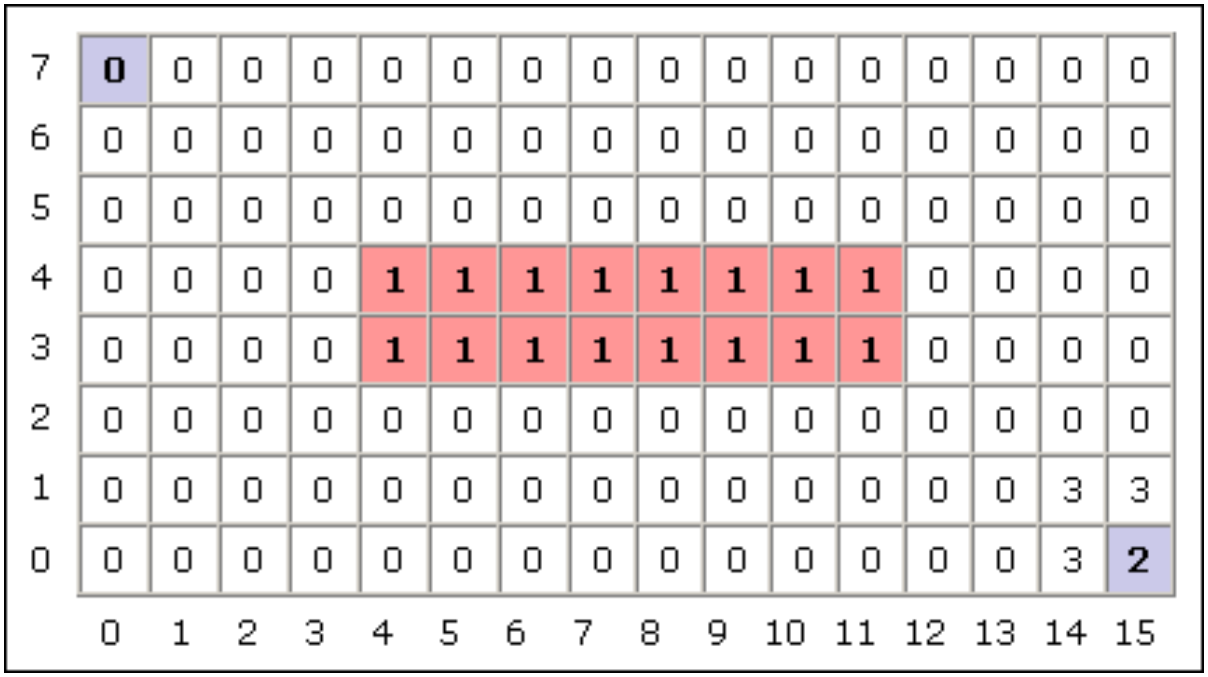


The Wavefront Planner: Setup

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**2**

The Wavefront in Action (Part 1)

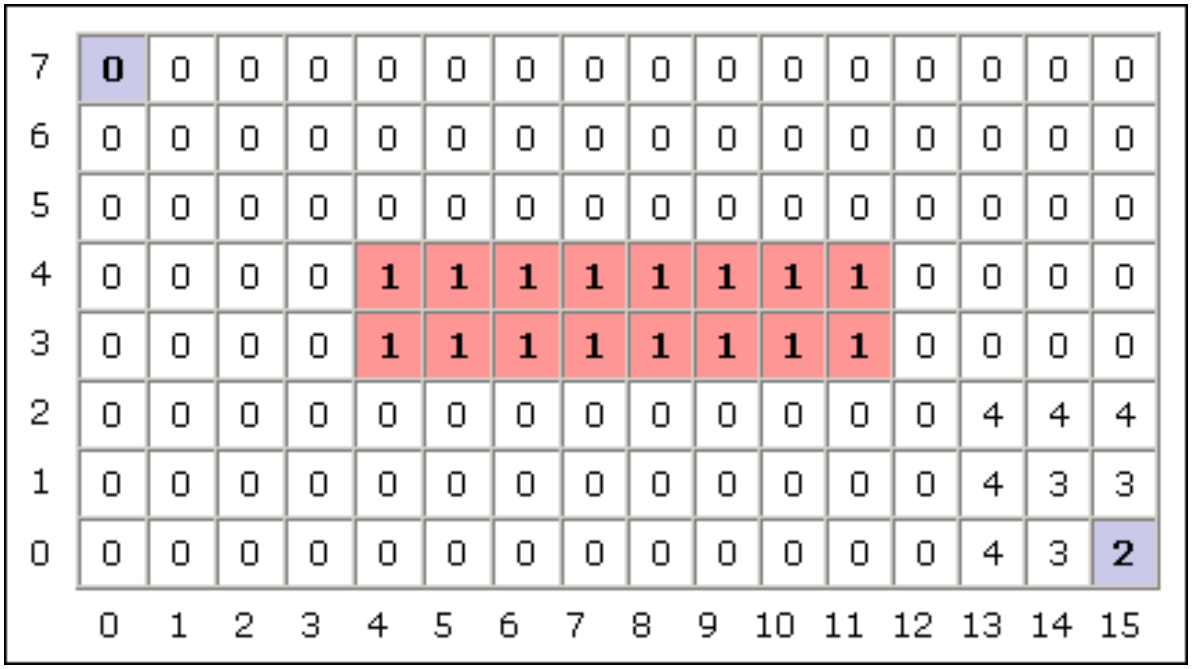
- Starting with the goal, set all adjacent cells with “0” to the current cell + 1
  - 4-Point Connectivity or 8-Point Connectivity?
  - Your Choice. We’ll use 8-Point Connectivity in our example



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	**2**

The Wavefront in Action (Part 2)

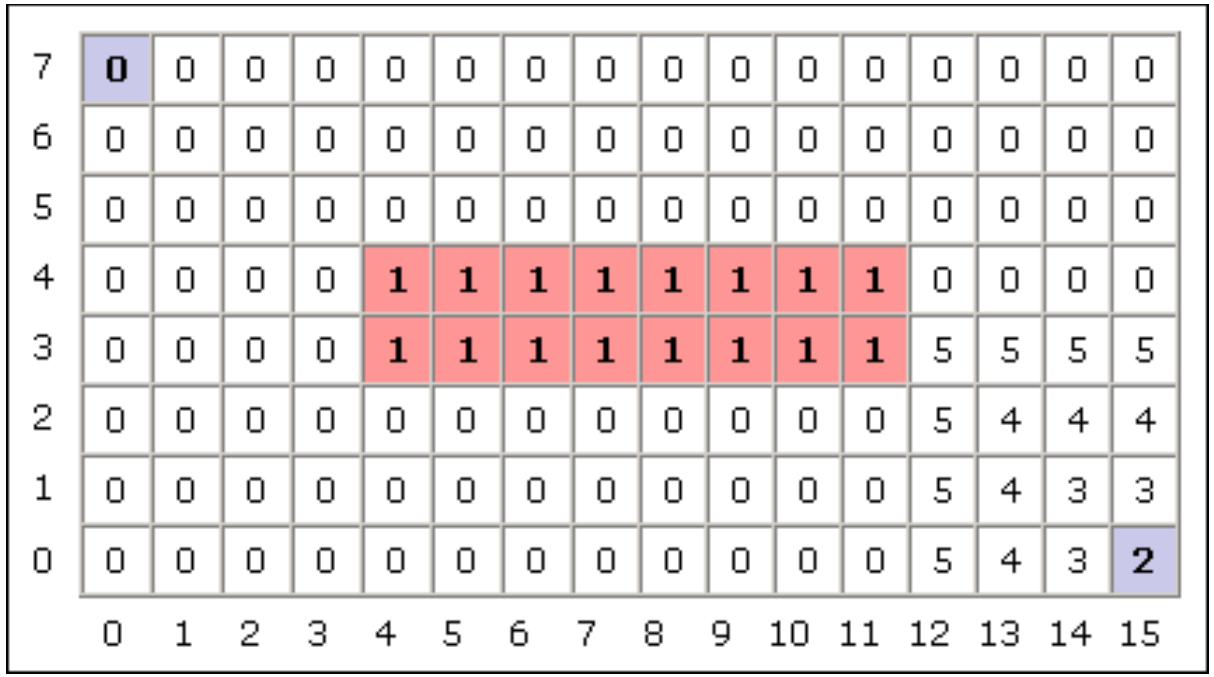
- Now repeat with the modified cells
  - This will be repeated until no 0's are adjacent to cells with values >= 2
    - 0's will only remain when regions are unreachable



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4
1	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	**2**

The Wavefront in Action (Part 3)

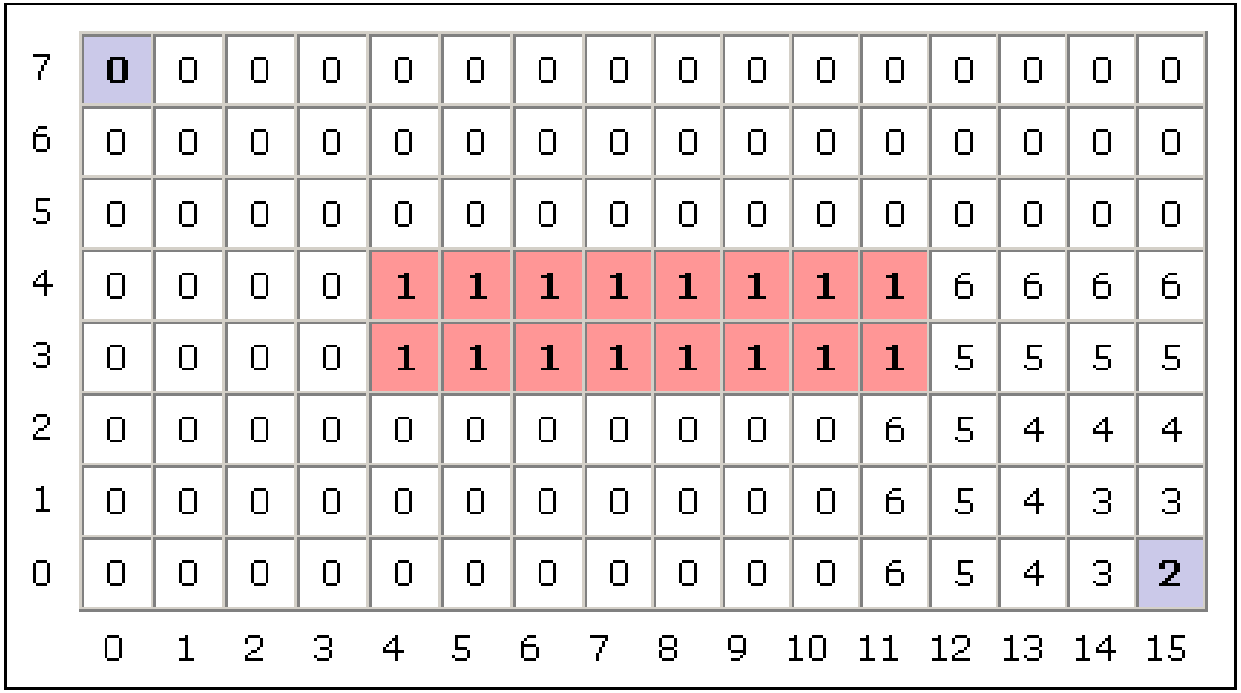
- Repeat again…



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	5	5	5	5
2	0	0	0	0	0	0	0	0	0	0	0	0	5	4	4	4
1	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3	3
0	0	0	0	0	0	0	0	0	0	0	0	0	5	4	3	**2**

The Wavefront in Action (Part 4)

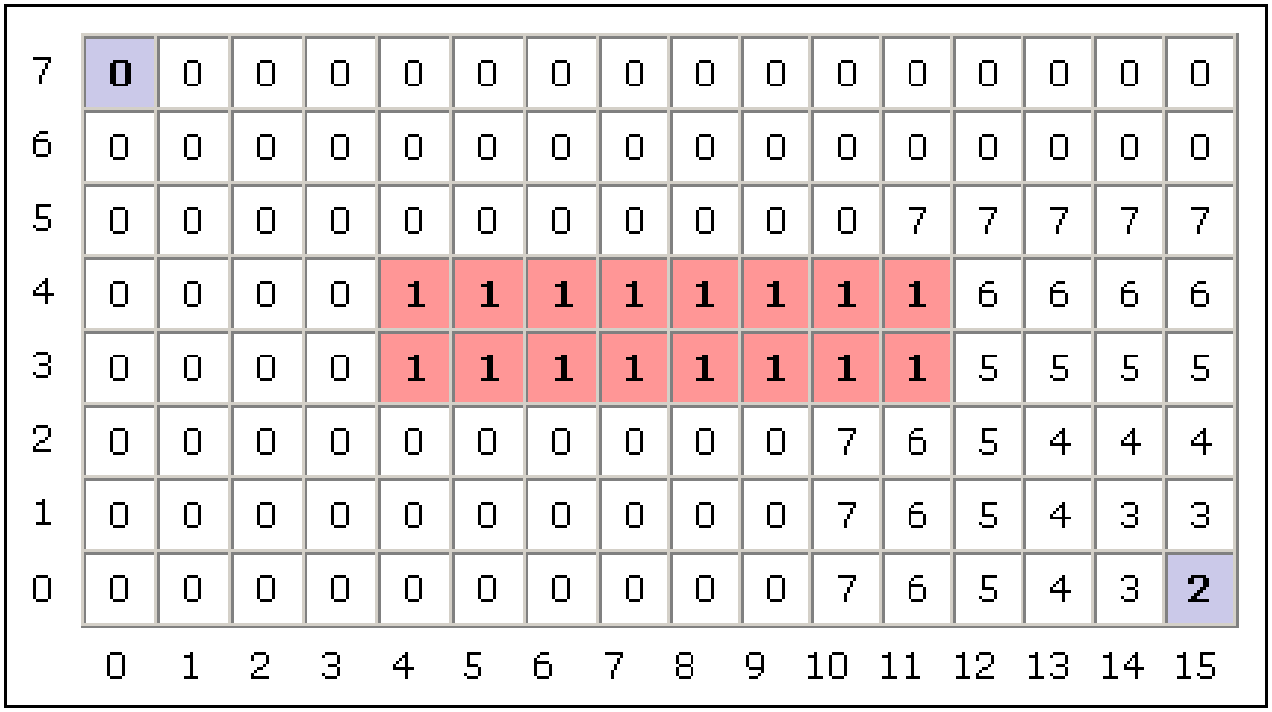
- And again…



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	6	6	6	6
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	5	5	5	5
2	0	0	0	0	0	0	0	0	0	0	0	6	5	4	4	4
1	0	0	0	0	0	0	0	0	0	0	0	6	5	4	3	3
0	0	0	0	0	0	0	0	0	0	0	0	6	5	4	3	**2**

The Wavefront in Action (Part 5)

- And again until…



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7	**0**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	7	7	7	7	7
4	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	6	6	6	6
3	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	5	5	5	5
2	0	0	0	0	0	0	0	0	0	0	7	6	5	4	4	4
1	0	0	0	0	0	0	0	0	0	0	7	6	5	4	3	3
0	0	0	0	0	0	0	0	0	0	0	7	6	5	4	3	**2**

The Wavefront in Action (Done)

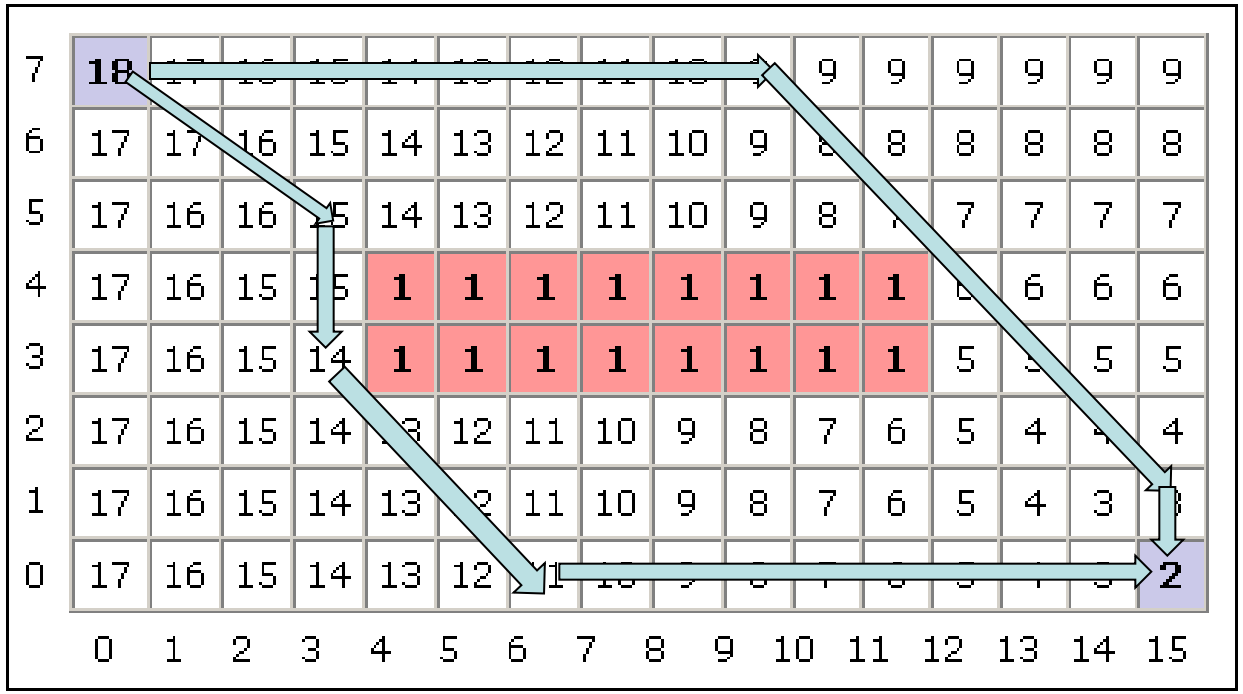
- You're done
  - Remember, 0's should only remain if unreachable regions exist

7	**18**	17	16	15	14	13	12	11	10	9	9	9	9	9	9	9
6	17	17	16	15	14	13	12	11	10	9	8	8	8	8	8	8
5	17	16	16	15	14	13	12	11	10	9	8	7	7	7	7	7
4	17	16	15	15	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	6	6	6	6
3	17	16	15	14	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	5	5	5	5
2	17	16	15	14	13	12	11	10	9	8	7	6	5	4	4	4
1	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	3
0	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	**2**
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The Wavefront, Now What?

- To find the shortest path, according to your metric, simply always move toward a cell with a lower number
  - The numbers generated by the Wavefront planner are roughly proportional to their distance from the goal

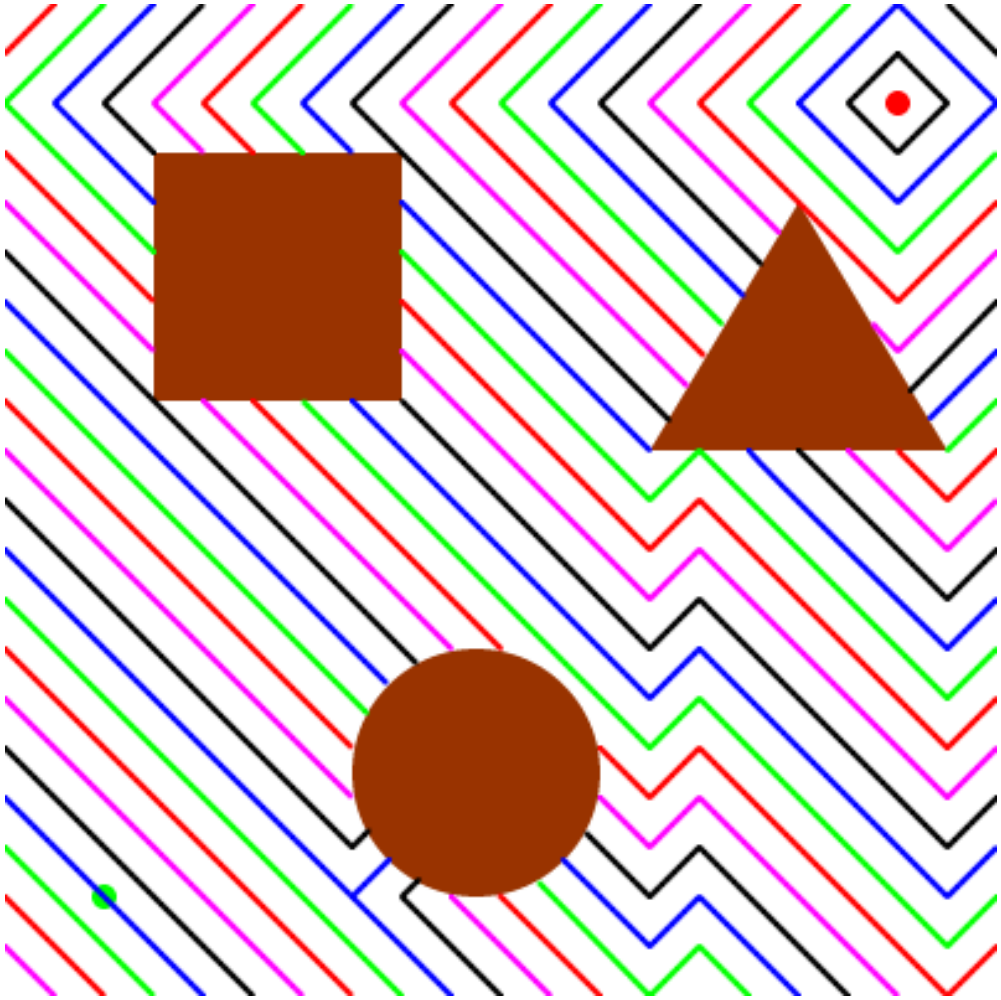
Two possible shortest paths shown



Wavefront (Overview)

- Divide the space into a grid.
- Number the squares starting at the start in either 4 or 8 point connectivity starting at the goal, increasing till you reach the start.
- Your path is defined by any uninterrupted sequence of decreasing numbers that lead to the goal.

Wavefront Slide



Wavefront References

- http://www.societyofrobots.com/programming_wavefront.shtml

Final Notes

- Introduction to path planning challenge
  - Motion capture, grid based, obstacles, map vs. no map