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LINEAR ALGEBRA

PRE-MID ASSIGNMENT

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BSSE 4 | ROLL NO: 15126

Cramer's Rule

Solving a physical system of linear equation by using Cramer's rule

Cramer's rule is really useful for the solution of any given system of Linear Equations. By using this rule, we can solve any particular variable given in linear equation instead of solving the whole equation. Usually this rule is not explained in accordingly but the main purpose behind this rule is to solve a particular variable as given in any system of linear equations.

Example:

We will demonstrate Cramer's rule with the following system:

x + 2y + 3z = 1

-x + 2z = 2

-2y + z = -2

Step 1:

The coefficient matrix of this system is

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & -2 & 1 \end{pmatrix}$$

Note that the matrix is square (it has 3 rows and 3 columns), and so we may proceed with the next step of Cramer's rule.

Step 2:

Now find the determinant of the coefficient matrix A; use the matrix manipulator in the tools box if you would like help in this computation. You should get |A| = 12. This is not zero, so Cramer's rule may be applied here.

Step 3:

$$A_x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 2 \\ -2 & -2 & 1 \end{pmatrix}$$

and its determinant is $|A_x|$ = -20. Therefore x = $|A_x|$ / |A| = -20/12 = - 5/4.

Step 4:

Using the same method, the values for the remaining 2 variables, x and y, are computed below:

$$A_y = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 2 & 2 \\ 0 & -2 & 1 \end{pmatrix}$$

and its determinant is $|A_y| = 13$. Therefore $y = |A_y| / |A| = 13/12$.

$$A_z = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 0 & -2 & -2 \end{pmatrix}$$

and its determinant is $|A_z| = 2$. Therefore $z = |A_z| / |A| = 2/12 = 1/6$.

The main point is again the same that we did not needs to solve the whole system of linear equations to get the value of a single particular variable. In Cramer's rule, we needs to select any variable from a given system of linear equations and then apply Cramer's rule on it step by step and we will get its value as a result without solving the whole equation.

LU DECOMPOSITION

LU Decomposition is a matrix obtained as a result of product of Lower and Upper triangular Matrices. **LU** stands for **"lower upper"** and this solution system is also known as **LU factorization**. The LU decomposition was first time introduced by **Alan Turing** in **1948**.

LU Decomposition seems like the matrix form of **Guassian Elimination**. In computers, the the solution of **square systems of linear equations** is done by using the LU decomposition, and it is also a key step when inverting a matrix, or computing the determinant of a matrix.

Let *A* be a square matrix. An **LU factorization** refers to the factorization of A, with proper row and/or column orderings or permutations, into two factors, a lower triangular matrix *L* and an upper triangular matrix *U*,

$$A = LU,$$

In the lower triangular matrix all elements above the diagonal are zero, in the upper triangular matrix, all the elements below the diagonal are zero. For example, for a 3-by-3 matrix *A*, its LU decomposition looks like this:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

PROCEDURE:

Suppose we have to solve a linear system **Ax = b**, and that we can express the coefficient matrix **A** in the form of the socalled **LU decomposition A = LU**.

Then we may solve the linear system by the following procedure:

Stage l: Write **Ax = LUx = b**.

Stage 2: Set y = Ux, so that **Ax = Ly = b**. Use **forward substitution** on **Ly = b** to find $y_1, y_2, \ldots, y_n$ in that order, i.e., suppose the augmented matrix for the system **Ly = b** is:

$$\begin{bmatrix} \ell_{11} & 0 & \cdots & 0 & 0 & b_1 \\ \ell_{21} & \ell_{22} & \cdots & 0 & 0 & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \ell_{n-1,1} & \ell_{n-1,2} & \cdots & \ell_{n-1,n-1} & 0 & b_{n-1} \\ \ell_{n1} & \ell_{n2} & \cdots & \ell_{n,n-1} & \ell_{nn} & b_n \end{bmatrix}$$

Then forward substitution yields $y_1 = b_1/l_{11}$, and, subsequently,

$$y_i = \frac{1}{\ell_{ii}}\left[b_i - \sum_{j=1}^{i-1} \ell_{ij} y_j\right], \qquad i = 2, 3, \ldots$$

Note that the value of y_i depends on the values $y_1, y_2, \ldots, y_{i-1}$, which have already been calculated.

Stage 3: Finally, use **back-substitution** on **Ux = y** to find $x_n, \ldots, x_1$x in that order.

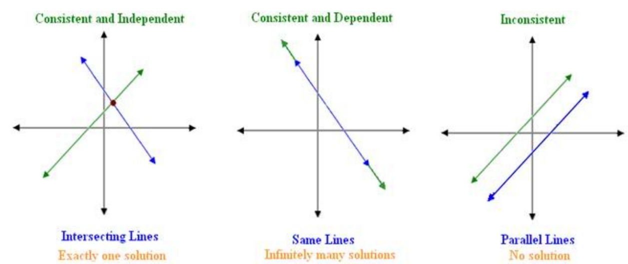
Later on we shall outline a general method for finding **LU** decompositions of square matrices. The following example shows this method in action, involving the matrix **A = L₁U₁**above. If we wish to solve **Ax = b** for a number of different **b**s, then this method is more efficient than Gauss elimination. Once we have found an **LU** decomposition of **A**, we need only do forward and backward substitutions to solve the system for any **b**.

CONSISTENT AND IN - CONSISTENT SYSTEM OF LINEAR EQUATIONS

In mathematics, a system of linear equations is a collection of two or more linear equations with the same set of variables in all the equations.
In other words, we can say a system of linear equations is nothing but two or more equations that are being solved simultaneously.

Mostly, the system of equations can be used by the business people to predict their future events. They will model a real world situation in to system of equations to find the solution and manage their business. We can make an accurate prediction by using system of equations.

The solution of the system of equations is an ordered pair that satisfies each equation in the system.



Consider the two equations,

$x + y = 2$

$x - y = 2$

It forms a system of equations in two variables. The solution of this system is the ordered pair (x, y).

Consider the equations given below.

$-3a + 2b - 6c = 6$

$5a + 7b - 5c = 6$

$a + 4b - 2c = 8$

It forms a system of equations in three variables. The solution of this system is the ordered pair (a, b, c).

The solution of a system of linear equations can be of three types. They are:-
(i) One solution (ii) Infinite solution and (iii) No solution

If the graph of the equations intersects each other at a point, then ordered pair corresponding to the point of intersection will be the solution to that system. In this case, the system will have exactly *one solution*.

If a system has exactly one solution, then the equations are said to be*independent*.
If the graph of the equations coincides, then all the points on the line will be the solution to that system. In this case, the system will have *infinite number of solutions*.

If the system has infinite number of solutions, then the equations are said to be *dependent*.
If the graphs of the equations are parallel, then the system of equations will have *no solution*. Because parallel lines never intersect each other.
If the system has at least one solution (one solution or infinitely many solutions), then it is said to be *consistent system*.
If the system has no solution, then it is said to be *inconsistent system*.
Both types of equation system, consistent and inconsistent, can be any of overdetermined (having more equations than unknowns), underdetermined (having fewer equations than unknowns), or exactly determined

TRIVIAL AND NON-TRIVIAL SOLUTIONS

A vector is called **trivial** if all its coordinates are 0, i. e. if it is the zero vector. In Linear Algebra we are not interested in only finding one **solution** to a system of linear equations.

TRIVIAL:

While, A **solution** or example that is not **trivial**. Often, **solutions** or examples involving the number zero are considered **trivial**.

NON-TRIVIAL:

Nonzero **solutions** or examples are considered **nontrivial**. For example, the equation x + 5y = 0 has the **trivial solution** (0, 0).

THE END